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M. Garcia-Fernandez
M. Hernandez-Pajares
J. M. Juan
J. Sanz

Performance of the improved Abel transform to estimate electron density profiles from GPS occultation data

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M. Garcia-Fernandez (✉)
Research Institute for Sustainable
Humanosphere (RISH), Kyoto University,
Uji Campus, Gokasho,
Uji, Kyoto 606-0011, Japan
E-mail: mgarcia@rish.kyoto-u.ac.jp
Tel.: +81-774-383825
Fax: +81-774-318463

M. Garcia-Fernandez
M. Hernandez-Pajares · J. M. Juan
J. Sanz
Research group of Astronomy
and Geomatics (gAGE), Applied Math.
Dept. IV, Technical Polytechnic University
(UPC), Jordi Girona 1-3, mod C3, Campus
Nord 08034, Barcelona, Spain
E-mail: manuel@mat.upc.es
E-mail: miguel@fa.upc.es
E-mail: jaume@mat.upc.es

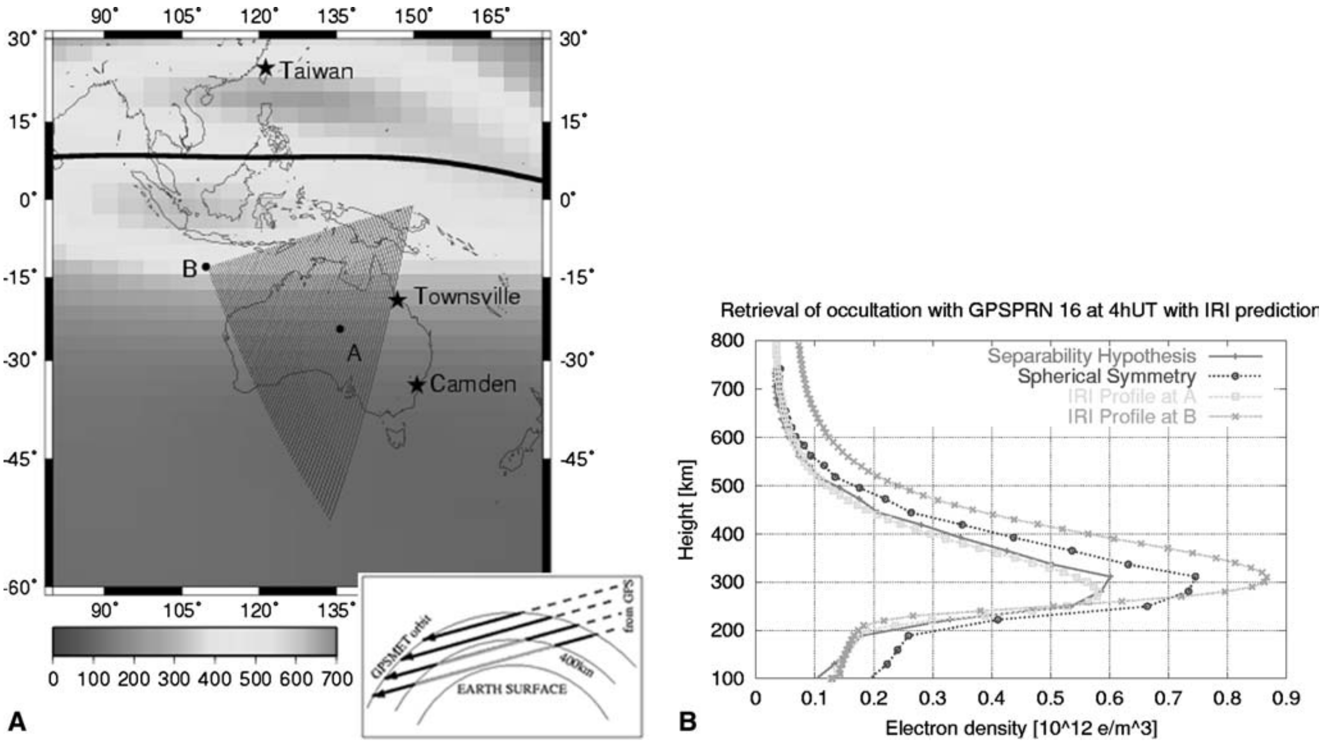
Abstract The Abel inversion is a straightforward tool to retrieve high-resolution vertical profiles of electron density from GPS radio occultations gathered by low earth orbiters (LEO). Nevertheless, the classical approach of this technique is limited by the assumption that the electron density in the vicinity of the occultation depends only on height (i.e., spherical symmetry), which is not realistic particularly in low-latitude regions or during ionospheric storms. Moreover, with the advent of recent satellite missions with orbits placed around 400 km (such as CHAMP satellite), an additional issue has to be dealt with: the treatment of the electron content above the satellite orbits. This paper extends the performance study of a method, proposed by the authors in previous works, which tackles both problems using an assumption of electron-density separability between the vertical total electron content and a shape function. This allows introducing horizontal information into the classic Abel inversion. Moreover, using both positive and negative elevation data makes it feasible to take into account the electron content above the LEO as well. Different data sets involving different periods of the solar cycle, periods of the day and satellites are studied in this work, confirming the benefits of this improved Abel transform approach.

Introduction

Abel inversion has been applied in the last decade to provide vertical profiles of ionospheric electron density with a high degree of accuracy (Hardy et al. 1993; Hajj et al. 1998; Schreiner et al. 1999). The expected performance of this technique, when compared with ionosonde data, varies from 20% to 40% in the estimation of the NmF2 or peak of electron density of the F2 layer (or equivalently from 10% to 20% in the case of F2 layer critical frequency). Nevertheless, the main drawback of the classical Abel inversion approach is the assumption of spherical symmetry, which assumes that the electron density depends only on height in the vicinity of the occultation. To overcome this limitation, Hernandez-Pajares et al. (2000) introduced the concept of electron density separability between the vertical total electron content (VTEC) and a shape function. This approach considers the VTEC horizontal variation to describe the electron density variation, preserving the iterative nature of the classic Abel transform. The feasibility of the separability hypothesis was first shown in Hernandez-Pajares et al. (2000) using both simulated data from the IRI model (Bilitza 1990) and real data. Garcia-Fernandez et al. (2003) showed how the use of shape functions diminished the effect of the co-location distance on the comparison with ionosondes. This result showed that the geographical variation of shape functions is lower than the electron density profiles. The improvement (in a qualitative way) of the separability hypothesis approach can be seen in Fig. 1.

An additional issue that has to be taken into account is the electron content above the LEO. In the case of satellites such as GPSMET or SAC-C, with respective orbital heights of 750 and 700 km, the upper ionosphere and plasmasphere contribution is small enough to be considered as a correction, for instance with an exponential extrapolation (Hajj et al. 1998; Hernandez-Pajares et al. 2000). However, with very low earth orbiters such as CHAMP (with initial orbital height of 450 km), this extrapolation may not fully account for the contribution of the electron content above the LEO.

Fig. 1 Typical GPSMET occultation near Australia. The parts of the ray with height between GPSMET orbit and 400 km are marked with *dark gray* and those parts of the ray with heights under 400 km with *light gray*. The Appleton–Hartree anomalies (also shown near the thick line representing geomagnetic equator) take place in the occultation occurrence, thus making the spherical symmetry hypothesis especially unrealistic. In this case, the variation of the VTEC within the occultation region may reach values closer to 50 TECU. The figure also shows the set of profiles corresponding to the points A and B in the map simulated with the IRI model (Bilitza 1990). The profiles show how large horizontal gradients of electron density are present in the occulting region. The spherical symmetry retrieval offers a solution between both profiles, while the shape function (defined in the *Inversion Scheme* section) retrieved with the separability hypothesis when scaled by the appropriate VTEC (in the figure it has been scaled by the VTEC at A) matches the corresponding IRI profile at point A



In this work, the method described in Hernandez-Pajares et al. (2000) and Garcia-Fernandez et al. (2003) is applied to process radio occultations gathered by the satellites CHAMP and SAC-C. A typical comparison with the ionosonde measurement of F2 layer critical frequency (i.e., foF2) is performed to test the Abel inversion profiles. For testing the performance of the F2 layer peak height (i.e., hmF2), the Dudeney formula (Dudeney 1983) is used in order to extend the number of comparisons.

Inversion scheme

Following the idea of *separability* of the electron density between the VTEC and the electron density through a shape function (m^{-1}) introduced in Hernandez-Pajares et al.(2000) and later developed further in Garcìa-Fernandez et al. (2003), the electron density may be expressed as:

$$N_e(\lambda, \phi, h) = \mathrm{VTEC}(\lambda, \phi) \cdot F(h) \quad (1)$$

Note that the VTEC provides the horizontal variation and the shape function describes the vertical variation of the electron density. This hypothesis allows reformulating the classical Abel inversion in its discrete form by substituting the VTEC and shape function for the electron density. Therefore, the GPS LI observable may be easily expressed as:

$$L_I(p_j)=b_I+\alpha\cdot\Delta l_p\cdot F_p\cdot \mathrm{VTEC}(\lambda_p,\phi_p)$$
$$+\alpha\cdot\sum_{k=0}^{j}\Delta l_{jk}\cdot F(p_k)\cdot[\mathrm{VTEC}(\lambda_k,\phi_k)+\mathrm{VTEC}(\lambda_{k'},\phi_{k'})]. \quad (2)$$

A schematic view of this expression is depicted in Fig. 2.

The VTEC at each geographical location ($\lambda_{i,\phi i}$) can be retrieved from the global ionospheric maps of the international GPS service given in IONEX format (Schreiner et al. 1999) or any other external VTEC model. The constant α is 1.05 $m_{LI}/10^{17}$ electron m^{-2} . The unknowns are the bias b_I (which contains both the instrumental biases and the phase ambiguity), the shape functions (which describes the ionospheric profile under the LEO orbit $F(p_k)$), and the shape function element F_p (which accounts for the electron content above the LEO satellite). The use of both positive and negative elevation data gathered by the LEO GPS receivers allows distinguishing between the b_I and the F_p variables.

In this work, the STEC has been obtained from the geometric free combination. Hajj et al. (2000) and Syndergaard (2002) recommend the use of TEC computed directly the L1 carrier phase observable (or a geometric combination of L1 and L2) corrected for nondispersive terms in order to diminish the bending effect (the departure of the ray from the straight line) that can reach up to 20 TECUs (in an occultation that has a maximum TEC value above 2000 TECUs). However, Schreiner et al. (1998, 1999), who applied the classical Abel inversion, did not find significant differences between the uses of such different TECs when they estimate the foF2.

The technique proposed in this work can be applied in both cases, but the use of the geometric free combination has been chosen because of its simplicity. See Schreiner et al. (1998) for details how the L1 TEC estimation should be implemented. The effect of the bending error that depends on the electron density gradients is small. It would introduce an overestimation of the top layers and, as a consequence, a reduction of the F peak which would occur higher (up to 5 km, Hajj et al. 2000).

foF2 and foE estimation

In order to quantify the performance of this technique and compare it with the classical spherical symmetry assumption, ionosonde measurements are used as a standard for comparison to electron density profiles estimated with LEO occultation data. The data set consists of observations from three different LEO missions: (1) GPSMET, with observations during October 1995 (Solar Minimum), (2) SAC-C and (3) CHAMP. Both SAC-C and CHAMP data correspond to November 2002 (solar maximum).

The comparisons are made to ionosondes that were mainly placed at midlatitudes (Europe, North America and Australia). To discard doubtful values of either VTEC from IONEX or NmF2 from ionosondes, a filter on the slab thickness (or ratio between the VTEC and the NmF2) has been used. The idea of this filter, when applied to ionosondes, is to delete unrealistic ionosonde foF2 measurements, mainly during night, that can be seen as clear discontinuities in the foF2 estimation. This is necessary in order to have confident "truths". On the other hand, when applied to occultations, the filter deletes automatically bad estimations of electron density profiles. These bad estimations are due to the LEO data: (1) the LEO can be below the foF2 maximum where it is not possible to estimate the value of the F peak; (2) cycle slips can occur in low-rate receivers that cannot be distinguished from real TEC variation (this can introduce jumps in the profile estimation affecting the estimation of

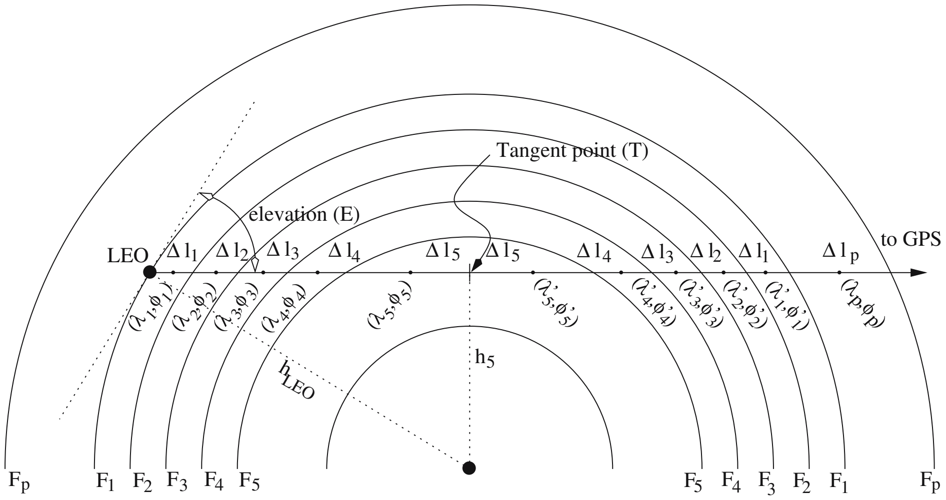


Fig. 2 Schematic view of GPS LI modeling with shape functions (*F*). The VTEC at the locations of each layer midpoint is known. A *straight-line* approximation is assumed

bottom layers). This filter discarded ionosonde measurements and occultation profiles with slab thickness under 175 km and above 1,000 km, which represents around 10% of the total number of comparisons. Of these, 70% or more are due to unrealistic ionosonde measurements and the remaining percentage (i.e., 3% of all comparisons) is due to bad retrieved profiles for the GPSMET and SAC-C satellites. In the case of the CHAMP satellite, the number of rejected occultations shows an increase of 50% due to the fact that some of the profiles may start under the F2 peak, making the inversion algorithm not applicable for these cases. Note that, by Eq. 1, the separability hypothesis assumes a constant slab thickness in the region and duration of the occultation, but extreme values of this parameter can account for questionable ionosonde measurements. Therefore, the outliers of this parameter have been discarded as well as abrupt changes in time. Table 1 summarizes the errors in the estimation of the maximum of the F2 layer.

It can be seen that in all cases that the results assuming the separability hypothesis offer an improvement, when compared to those obtained with spherical symmetry. The average improvement is approximately 30%. For the ionosonde observations during solar maximum (periods for CHAMP and SAC-C), the variation of the slab thickness is quicker and noisier than minimum solar activity (period of GPSMET), specially in periods outside the daytime. This seriously affects the performances during this period, as shown in Table 1. With the separability hypothesis, this degradation of the results can be diminished with the aid of VTEC information. The improvement of separability on these night periods is approximately 25%. Similar improvement is also confirmed in the foF2 intercomparisons between CHAMP and SAC-C profiles, where results showed a similar level of agreement with respect to those obtained with ionosondes (Garcia-Fernandez et al. 2004). Note that the occultation can provide a measurement of the slab thickness by inverting the shape function at the hmF2 (see Eq. 1). This assumption of constant slab thickness in the occultation occurrence is vindicated to a great extent during daytime, but certain excursions of this value are experienced during dawn and dusk periods. Results show that, in addition to being better than the spherical symmetry approach, the separability hypothesis is also able to cope, to a more limited extent, with these variations.

Figure 3 shows two particular cases of this point. The A panel shows an example in which the VTEC and NmF2 behave similarly (slab thickness constant), thus providing good comparisons. On the other hand, when they behave differently (B panel, slab thickness not constant), the comparisons worsen.

Comparing the results for the foF2 estimation for SAC-C and CHAMP, only a slight worsening is seen in the case of CHAMP (the lowermost LEO). This confirms that the strategy for the topside electron density estimation, which affects mainly the CHAMP observations, is sufficient to describe the contribution of the TEC above both LEOs.

The improvement of this technique can also be seen in the estimation of the E layer. Since the Abel inversion is a technique that starts at the uppermost part of the profile and processes downward, the errors in the upper layers can dramatically affect the lower layers, especially the E layer. The separability hypothesis improves the overall estimation of the profile. This is confirmed by an improvement on the lower layer. This improvement is better than 40% with respect to the spherical symmetry approach (see Table 2 for the performance in the E-layer estimation using GPS/MET radio occultations).

Table 1 The absolute errors (in MHz) and relative errors of the estimations of foF2 using different satellites

LEO	Period	Separability hypothesis			Spherical symmetry
		#comp	Absolute error (MHz)	Relative error (%)	Relative error (%)
GPSMET at 750 km	Day	2231	0.75	12.2	18.3
(10th–21th Oct.1995, Solar minimum)	D&D	918	0.80	19.0	25.7
	Night	1222	0.65	19.0	25.0
SAC-C at 700 km	Day	5868	1.29	12.5	19.6
(1st–16th Nov. 2002, Solar maximum)	D&D	529	0.97	18.1	27.6
	Night	1946	1.29	25.4	41.4
CHAMP at 420 km	Day	1966	1.40	14.4	24.3
(same period as SAC-C)	D&D	189	1.14	21.5	42.0
	Night	879	1.55	28.4	54.6

The relative errors regarding the spherical symmetry approach are given as well. Different day periods are considered: Daytime, Dawn and Dusk and Night. The co-location distances between the occultation and ionosonde has been set up to 2,000 km and the comparisons are made using a time window of 1 h centered in the occultation occurrence. The first week of November was geomagnetically more active than the second one, but no substantial difference in the statistics has been found between them, therefore, the statistics of CHAMP and SAC-C contain both weeks

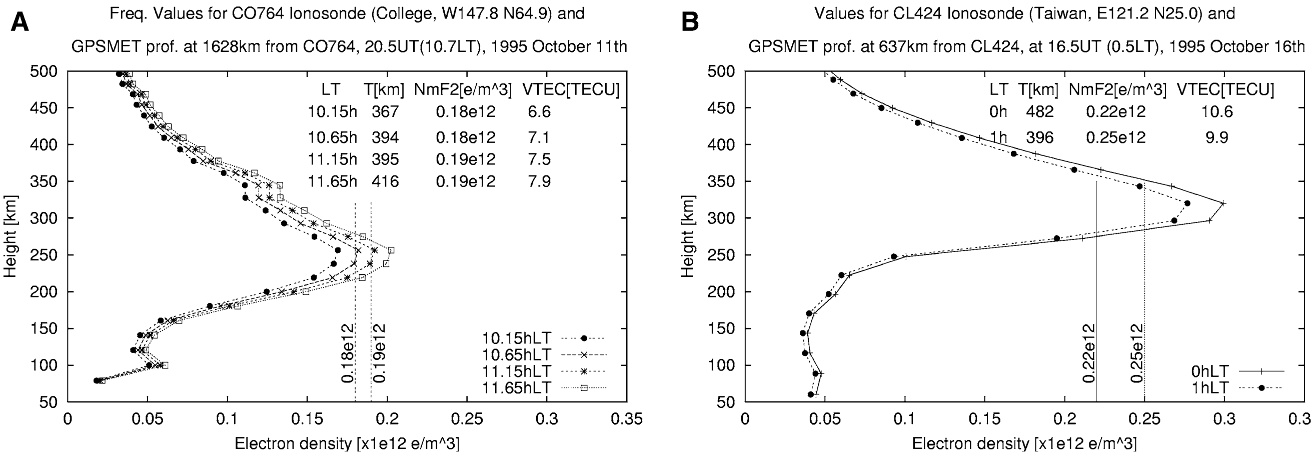


Fig. 3 Effect of slab thickness (*T*) in the comparison between foF2 measurement by ionosonde and profiles retrieved from GPSMET data. Each panel contains the corresponding shape function multiplied by the VTEC at the time when the comparison with ionosonde was made. Panels include the slab thickness parameter, the NmF2 value and VTEC over the ionosonde for different local times. (**A**) panel corresponds to a case of (almost) constant value of slab thickness while (**B**) panel offers the opposite case in which the NmF2 increases but the VTEC decreases. (Figure has been reproduced by permission from the American Geophysical Union)

hmF2 estimation

Comparing real heights from Abel inverted profiles with ionosonde data has not been as widely used as the frequency comparisons because no direct measurements of hmF2 are possible with ionosondes. It is necessary to rely on techniques, such as POLAN (Titheridge 1998), that converts the ionosonde data from virtual heights to real heights, or more straightforward methods that allow estimation of the hmF2 parameters using the M(3000)F2 parameter and the ratio between the critical frequencies of both F2 and E layers (Dudeney 1983).

The wide availability of the M(3000)F2 values as well as the F2 and E-layer critical frequencies makes it possible to obtain a large number of hmF2 parameter estimates. In order to obtain reliable values for the hmF2, other researchers, such as Rishbeth et al. (2000), apply the following constraints: (1) the M(3000)F2 must be larger than 2.5 and (2) the ratio foF2/foE must be greater that 1.7. This work follows the same criteria to obtain the values of the F2 layer peak height. The accuracy of this method, with respect to direct measurements, is between 20 km and 30 km RMS (Zhang et al. 1999). Table 3 shows the comparison of Abel inversion using the separability hypothesis with the hmF2 values obtained with the Dudeney formula for the same periods as in the cases of the foF2 estimations. Note that, in the GPSMET cases, the bias is larger than in the CHAMP and SAC-C cases. This could be due to a certain local time effect. In the case of GPSMET, the transition north to south took place at noon, when the highest electron density variation occurs. This affects the separability assumption since just one shape function cannot cope with large variations in height of the electron density distribution. This effect is reduced in the cases of CHAMP and SAC-C since the transitions south to north took place before 10 h LT.

The discrepancies between the hmF2 estimates are between 20 km and 30 km and are similar regardless

Table 3 The performances corresponding to the comparison of the hmF2 with the Dudeney formula. Co-location distance between occultation and ionosonde is also set up to 2,000 km and time interval centered at occultation occurrence is set up to 1 h as well

	#comp	hmF2 bias (km)	hmF2 σ (km)
GPSMET	2,457	−8.55	25.57
SAC-C	3,141	−2.4	28.8
CHAMP	1,812	0.7	33.3

Table 2 The table includes the comparison with the E- and Es layers. The statistics includes the RMS in absolute values (in MHz) and relative percent values for both separability and spherical symmetry cases. The co-location distance between occultation and ionosonde is set up to 2,000 km and time interval centered at occultation occurrence is set up to 1 h as well

GPS/MET	#comp	Separability hypothesis RMS: MHz[%]	Spherical symmetry RMS: MHz[%]
E layer	135	0.4[17.1]	0.7[28.5]
Es layer	35	0.5[16.2]	1.0[30.4]

of the satellite used. This result is in agreement with the comparisons involving the separability hypothesis. Since the differences are almost equal in both the separability and spherical symmetry cases, only those corresponding to the former are shown in Table 3. The discrepancies shown in Table 3 are similar when an intercomparison is made between inverted profiles of CHAMP and SAC-C (Garcia-Fernandez et al. 2004). For this profile comparison, an average number of 30 km in the hmF2 estimation has also been obtained.

Conclusions

This work confirms that the introduction of VTEC information into the classical Abel inversion scheme allows improved estimation of the vertical profiles of electron density at different local times and ionospheric conditions. This improvement is about 30% in the F2 layer peak estimation compared to the spherical symmetry approach. There is also an improvement in the E-layer estimation where a better estimation of the upper layers reduces the error of the lower layers of the electron density profiles.

Although the improvement is clear in the frequency domain and produces no large discrepancies in the estimation of real heights; the height estimates are similar to those obtained with the Dudeney formula which results in discrepancies between 20 km and 30 km. Comparison with CHAMP and SAC-C reveals that the topside estimation proposed in this work is accurate enough to offer similar results for both LEOs, being only slightly worse in the case of the CHAMP.

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