STUDY OF SEVERAL INTERPOLATION SCHEMES TO PROVIDE IONOSPHERIC CORRECTIONS FOR NAVIGATION

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Abstract

Since the development of space based geodesic techniques, the increasing of the positioning accuracy has been a major goal. In this context, the effect of the ionosphere has been a key issue to be taken into account to improve the navigation results obtained using several types of receivers: from dual frequency phase receivers at distances greater than tens of kilometres of the nearest fixed site to single frequency code receivers.
To overcome this problem, it is necessary either an accurate modelling of the ionosphere at the fixed stations and an optimum interpolation scheme for the rover receiver.
This approach is either valid for single frequency users or rovers using dual frequency phase receiver. The current work studies the performance of different interpolation schemes such as linear interpolation and kriging, which can be implemented in a rover receiver in order to interpolate either the Slant Total Electron Content (from now on STEC) or the Double Differenced STEC (∇?STEC) from the transmitted corrections that are computed at the fixed stations.

1. Introduction

With the advent of ambiguity resolution techniques it has made it possible to improve the accuracies of the Differential Global Positioning System (DGPS) from the meter level to up to a subdecimetric level. In order to do so, it is essential to take into account somehow the effect of the ionospheric delay, more concretely the value of the Double Differenced STEC (∇?STEC) between pairs of satellites and receivers. If the rover receiver and fixed site are near enough (baseline distances less than 20km) it can be applied the well-known technique Real Time Kinematic (RTK), that assumes, among other points, that the ∇ΔSTEC can be neglected due to the fact that the ionospheric refraction for the fixed station and rover are almost equal. Nevertheless this assumption is not realistic for longer baselines and/or in cases of local perturbations of the ionosphere such as Travelling Ionospheric Disturbances (TID). For these cases it is essential that the rover determines the ∇ΔSTEC with a precision better than ¼ TECU (2.7cm) [1]. In a given network of several fixed stations and a reference station (to which all differences are referred to), the fixed stations can compute their own ∇ΔSTEC from the GPS observables and send them to the rover, in such a way that after interpolation the rover may be able to solve adequately their own phase ambiguities. In [2] it was proposed a linear weighted interpolation scheme based on distance.
In this work it is suggested an alternative method based on kriging [3] and it is carried out a comparison with the distance weighted scheme. The kriging technique is widely used in geostatistics, but is has also proved useful for ionospheric estimation applied to navigation purposes (i.e. interpolate the Wide Area Augmentation System (WAAS) Ionospheric Corrections) as shown in [4]. In this work this interpolation technique, which takes into account the spatial correlation, is considered in order to obtain the ∇ΔSTEC at the rover location.
The results obtained in this work suggest that the best performance can be obtained when the interpolation scheme chosen is based on kriging techniques and applied directly on the ∇ΔSTEC of the reference stations, instead of using simpler interpolation schemes on the undifferenced STEC.

2. Method

The objective of the interpolation is to obtain a ∇?STEC accurate enough to allow the ambiguity resolution at the rover, i.e. with an error less than 2.7 cm, using the unambiguous STEC or ∇?STEC from the fixed stations which fulfil such requirement. Two different cases have been considered with a dataset affected by TID's gathered at mid-latitudes:

- Interpolation of the computed unambiguous STEC at the fixed stations. Once interpolated, the ∇?STEC is computed.
- Interpolation of the computed unambiguous ∇?STEC directly.

Once the ∇?STEC is estimated it is compared with the "truth" and it has been counted the number of occurrences in which the error was less than 2.7cm, allowing the real-time ambiguity resolution.
Since the interpolation scheme to the rover receiver becomes an important issue, two different interpolation methods have been considered to interpolate the data:

- Linear interpolation, with weights assumed to be $1/d^n$. (d: distance from the fixed stations to the rover receiver). This is the method used in previous references [2],[1]
- The kriging technique.

These approaches can be summarised in the following expression:

$$j_{rover} = \sum_i j_i \cdot w_i \qquad (1)$$

Where f_{rover} is the interpolated STEC or ∇?STEC at the rover receiver, f_i the unambiguous STEC or ∇?STEC at the fixed stations and $?_i$ the weight associated with each method. In this work the rover data observations of calculated unambiguous STEC and ∇?STEC do not take part into the interpolation scheme (as explained in [1] these values should be taken into account for scenarios with high gradients of Vertical TEC)

This work is focused in the interpolation scheme once the STEC or ∇?STEC are available. To obtain these values it is necessary to run in parallel a geodetic and ionospheric program in the stations. Since the coordinates for the fixed stations are accurately known, their ambiguities may be easily solved for errors in the ∇?STEC estimation of 1 TECU at 1σ. Therefore ambiguities can be solved with great degree of confidence [1], allowing an accurate estimation of the STEC. The "truth" of this work consists on considering BELL station as a fixed station (its coordinates are accurately known as well) and determining its STEC and ∇?STEC in conjuction with the reference stations. In the interpolation scheme the condition of "fixed station" for BELL is changed to "rover receiver" and their interpolated STEC and ∇?STEC are compared with the previously estimated.

2.1 Distance weighted linear interpolation

Usually, when it is necessary to interpolate the STEC or ∇?STEC at short distances, the interpolation method is based in a simple form of $1/d^n$. In this work it has been considered the same approach of [2] where n=1,2. Thus the weights are computed as follows:

$$w_i = \frac{\frac{1}{d_i^n}}{\sum_i \frac{1}{d_i^n}} \qquad (2)$$

Where d is the distance between the rover receiver and the fixed station.

Notice that the parameter n controls the correlation radii between the known data. It is obvious that for a two given n ($n_1 < n_2$) the correlation (and, as a consequence, the weights) decrease faster in the case of n_2 as the distance grows. Only the nearest values have significant weights.

2.2 Kriging based interpolation

As it can be seen, the linear interpolation that uses weights based on the inverse of the distance does not take into account any relationship between the data used in the experiment (i.e. for a given geometry or location the weights are basically constant). Geostatistics provides this relationship with the use of kriging techniques. These techniques take into account the spatial correlation between the data, therefore, for a given geometry or location the weights can change as a function of the sample values [5]. The kriging method lies in the assumption that it is possible to compute a semivariogram, which, in fact, is a kind of correlation function that depends, on a first approximation, on the distance between the values. This semivariogram function may be expressed as:

$$\hat{g}(d_l) = \frac{1}{2 \cdot m(d_l)} \sum_{i \neq j}^{m(d_l)} \left(j_i - j_j\right)^2 \qquad (3)$$

Where $m(d_l)$ is the number of pairs at the bin-centered distance d_l, that corresponds to the set of pairs whose distance is between an interval centered at d_l. j_k corresponds to the values at the k-th sample known points at distance d_l.

Once the experimental semivariogram $\hat{g}$ is computed, it is necessary to adjust it to a theorical one from a set of theoretical semivariograms that must verify that are Conditional Negative Semidefinite. In fact, the assumption is that the variance is always positive [5]. In this case, a simple linear [6] semivariogram fits reasonable well:

$$g = a \cdot d + b \tag{4}$$

Being *a, b* adjustable constants by means of least square fitting.

Once the semivariogram is computed and adjusted from a set of initial data, it is easy to demonstrate the kriging equations (see details in [5]) only taking into account that kriging is a minimum-mean-squared-error method of spatial prediction that depends on the second order properties of the data distribution. In fact, kriging is the *Best Unbiased Linear Estimator (BLUE)* under these second-order proprieties, which allows computing some parameter with a minimum variance. Thus, the interpolated values can be computed as a linear combination (1) where its weights (*?* $_i$) are derived by the so-called kriging equations. Being ? the vector that contains the weights *?* $_i$:

$$q = \Gamma^{-1} \cdot \Gamma_0 \tag{5}$$

Where,

$$q = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \\ l \end{pmatrix} ; \Gamma = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1n} & 1 \\ g_{21} & g_{22} & \cdots & g_{2n} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nn} & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix} ; \Gamma_0 = \begin{pmatrix} g_{10} \\ g_{20} \\ \vdots \\ g_{n0} \\ 1 \end{pmatrix} \tag{6}$$

Where $?_{ij}$ is the theoretical semivariogram between the i-th and j-th locations (in the cases where data is available), and $?_{i0}$ is the theoretical semivariogram between i-th and the unknown locations. The vector $?_\mathrm{i}$ contains *?* $_i$, the weights used to interpolate, and *?* the Lagrange multiplier. From the eq. (5) and (6) it can be seen that sum of all *?* $_i$ must be 1. Nevertheless, this condition does not imply their positiveness [5].

3. Results

3.1. Scenario

The GPS network considered in this work is depicted in Figure 1. The fixed stations considered, that corresponds to the CATNET network of the Cartographical Institute of Catalonia (ICC), are Les Avellanes (AVEL), Montcada (MNTC), Llívia (LLIV) and Creus (CREU) and the reference station to which the double differences are referred to is Ebre (EBRE). The interpolation has been applied to the station Bellmunt (BELL), which has been considered as a rover. It has been studied during a period of 4 hours in the day June 11th 2001.

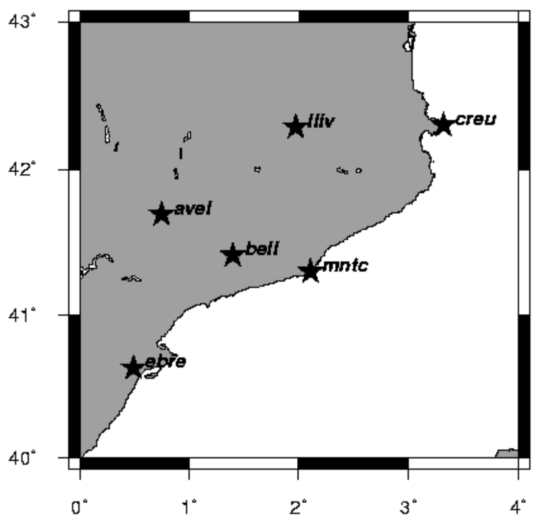

Figure 1: Map representing the reference stations (LLIV, MNTC, EBRE, AVEL, CREU) and the receiver (BELL) treated as rover in the experiment.

The minimum distance between the rover and the nearest fixed station is 62km (see table 1 for the other baselines), which is larger than the maximum distance required to perform RTK. Moreover, this period shows the presence of a TID as it can be seen in the ripples of the estimated vertical TEC and observed STEC variations (see Figure 2).

The navigation technique that offers subdecimetric errors in the position for baselines larger than 20km requires, typically, the use in parallel of a geodetic and ionospheric program at the reference stations. In this way, the phase ambiguities can be solved with a high degree of confidence, as mentioned above and was showed in [1]. Therefore the data set are the STEC (once the ambiguities are estimated and stabilised) determined by the stations and the ∇ΔSTEC are computed from this data fixing the reference station to Ebre. These values are considered as our "truth". Therefore, the computed STEC of the rover station (BELL) is compared with the "truth" of this station and the difference is what it has to be less than the threshold value of ¼ TECU (2.7cm) to be able to solve the carrier phase ambiguities.

STATION	DISTANCE TO BELL (km)
AVEL	62
MNTC	68
LLIV	108
EBRE	115
CREU	177

Table 1: Distances of the reference stations to the rover receiver (BELL).

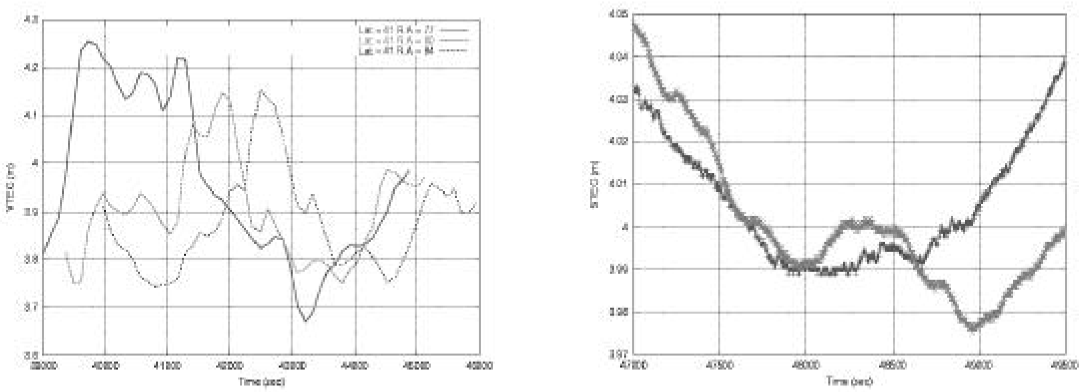

Figure 2: (a) TID observed with the vertical TEC computed for 41^{o} of latitude, at different local times, the ripples show the TID. (b) STEC observed by the two stations (at Barcelona and Roquetes) at a distance of 162 Km, the oscillations show the presence of TID as well.

3.2 How to obtain ÑDSTEC from STEC interpolation

In order to obtain ∇ΔSTEC using the estimation of the unambiguous STEC from the reference stations, the two interpolations schemes are applied to the data set of the experiment. In this case, the locations of the ionospheric pierce points (IPP) from each satellite is what is used for the computation of the distance to interpolate the undifferenced STEC values. In the two methods, the interpolation has been done for each satellite and each epoch because the behaviour of the STEC is different for each satellite, since the elevation and azimuth affect the value of the STEC. Thus, in the case that it is tested in this experiment, there are up to 5 IPP involved with a minimum elevation of 20^{o} to compute the STEC value at the rover receiver. Notice that it is possible to have less values of IPP depending on the performance of the reference station receivers.
In the case of the linear interpolation method based in $1/d^{n}$, the values of n=1,2 have been chosen, following (1) and (2) to compute $STEC_{rover}$. And then the ∇ΔSTEC has been computed and compared with the "truth" one (see Table 2 for details).
In the interpolation scheme based on kriging the steps applied are as follows:

1. First the semivariogram is computed. Due to the reduced amount of data points involved in the computation and the small associated statistical significance, the whole STEC data set is used, but only the data before the experiment are taken into account (see Figure 3). In this way, the computations still emulates the real-time mode. The final values for the adjusted semivariogram following (4), and the resulting parameters are $a = 4x10^{-3}$;b= 0.

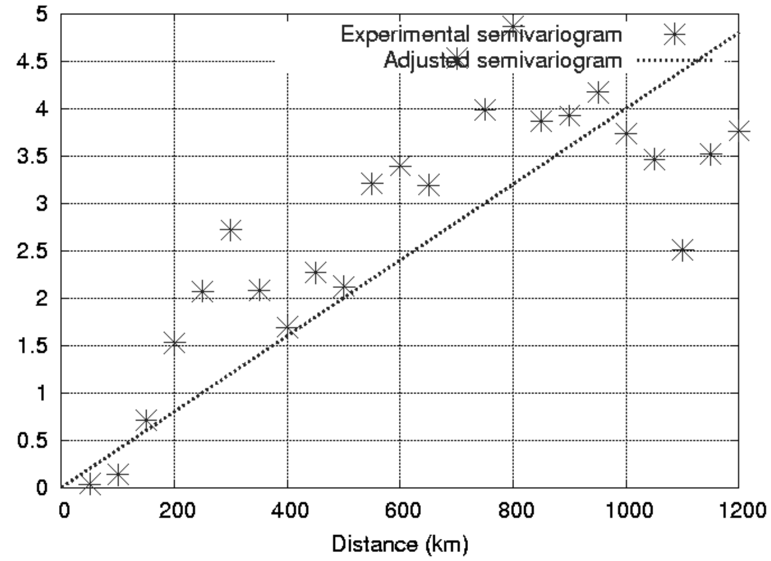

Figure 3: Experimental semivariogram (points) and the corresponding adopted theoretical semivariogram (line) for the STEC data.

2. Once the semivariogram is adjusted, the weights are computed following (5) and (6) and the $STEC_{rover}$ is obtained using the computed weights.
3. The last points deals with computing the STEC referred to a reference satellite and station in order to to compute the $\nabla\Delta$STEC and compared it with the “truth” (see Table 2).

3.3 How to obtain ÑDSTEC fromÑDSTEC interpolation

Other possibility studied in this work, is when the $\nabla\Delta$STEC are used, the roving user interpolates directly the $\nabla\Delta$STEC that are estimated by the fixed and reference station (up to 5 double differences corresponding to the stations of LLIV, AVEL, MNTC, CREU and EBRE). In the case of the reference station, notice that, by definition, the “$\nabla\Delta STEC_{EBRE}$” is equal to 0 and it has to be considered accordingly in the interpolation scheme.
To implement the algorithm based in $1/d^n$ to double differences is straightforward (see results in table 2), but when the interpolation scheme used is based in kriging, the number of different distance pairs are low due to the fact that the spatial reference for the double differences are the station coordinates. Therefore, the semivariogram has poor spatial resolution, such as in the case of undifferenced STEC. Even including the rover data, the semivariogram is not well defined and a linear semivariogram has been adjusted in order to obtain the corresponding weights (see Figure 4).

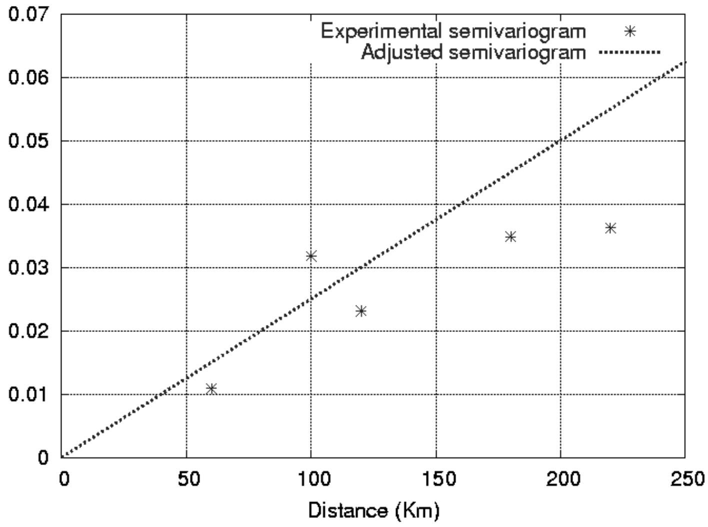

Figure 4: Experimental semivariogram (points) and the adjusted semivariogram (line) for the STEC data.

Notice that it has been choose a 25 Km bin interval for the semivariogram spatial data. To get a more reliable adjustment to the theoretical one. Then, the STECrover can be computed easily following (5) and (6) (see results at Table 2)

3.4 General Performance

To evaluate the different methods, the final interpolate ∇ΔSTEC at the rover receiver (BELL) has been compared with the "truth", computed as it has been mentioned above following [1]. To take into account the major significance of the results several statistics have been computed. First, the number of occurrences with an error below to 2.7 cm (¼ TECU) at the rover receiver is count (see Table 2) in order to determine which method has the best performance. Notice that to perform the statistics the number of unambiguous ∇ΔSTEC in the fixed stations is important, and in this case, it has been imposed that, at least, two unambiguous ∇ΔSTEC are available in the reference stations.

Interpolation scheme (since epoch 40000sec.)	Number of ∇ΔSTEC	Number of ∇ΔSTEC (with an error bellow 2.7 cm)	% of success
STEC (with 1/d)	519	390	75 %
STEC (with $1/d^2$)	519	409	79 %
STEC (with kriging)	519	443	85 %
∇ΔSTEC (with 1/d)	485	399	82 %
∇ΔSTEC (with $1/d^2$)	485	436	90 %
∇ΔSTEC (with kriging)	485	449	93 %

Table 2: Results for all the interpolation schemes used in the experiment.

As it can be seen in Table 2, the best performance is for the interpolation done with the ∇ΔSTEC using the kriging interpolation scheme with a performance of 93 % of success in the computation of the ∇ΔSTEC at the rover receiver, see figure 5 to see the error performance as a function of the elevation.

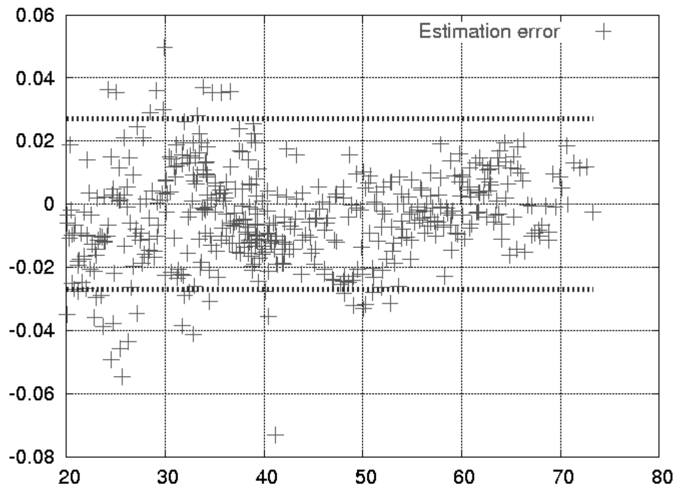

Figure 5: Estimation error of the ∇ΔSTEC, computed with kriging method, as a function of the elevation, the lines are the boundary of the 2.7 cm

It is possible to see in figure 5 that for high elevation, as it could be expected, the error decrease, but there are certain points that seem not to follow this trend. In fact, there are points that are affected by the interpolation error, related with the non-linear behaviour of the ∇ΔSTEC due to the TID's (see figure 6). This non-linear behaviour makes it difficult to interpolate the values at such distances of 60 Km or more, since, in particular, the solution can be out of the boundary delimited by the data.

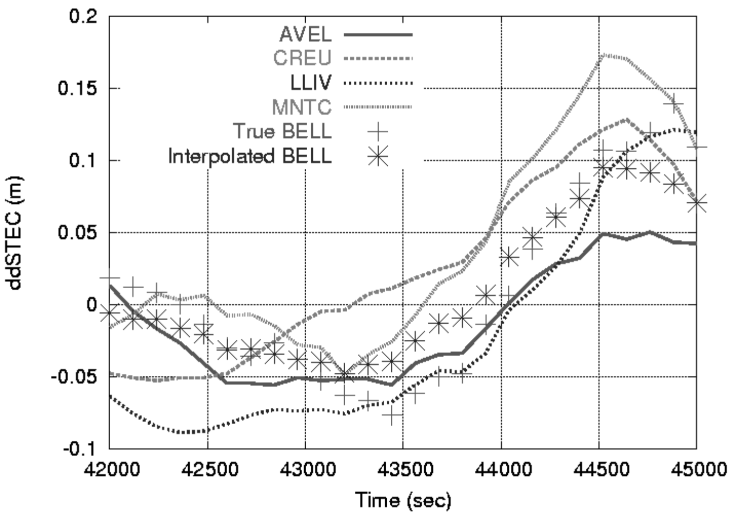

Figure 6: An example of no-linear behaviour of the ∇ΔSTEC at the rover receiver. It can be seen at the second 43500 (there are 4 values) where the true values of the rover (Crosses) are below to all the reference sites values. Also, the interpolated ∇ΔSTEC are depicted (asterisk).

4. Conclusions

The preliminary results shown in this work indicates that the use of geostatistic algorithm such as kriging techniques helps to improve the number of ambiguities solved in a rover receiver interpolating the accurate ∇ΔSTEC of the reference stations with an errors below 2.7 cm, due to the fact that kriging techniques uses the semivariogram that takes into account the spatial correlation between the data used. These techniques improve the results obtained with the approach that uses only the distance as a weighting scheme. An additional point to take

into account in the interpolation is considering directly the unambiguous ∇ΔSTEC helps to improve the ratio of occurrences in which the ∇ΔSTEC is interpolated with an error less than 2.7cm. Even without using the rover own data in the interpolation scheme in a scenario with a TID, the performance of the kriging technique remains acceptable in the context of the ambiguity resolution at distances of 60 km and more.
Future work can include a wider study in the same area of this work considering different geomagnetic conditions and longer baselines, to better characterize the performance of the kriging technique.

5. References

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