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Combining ionosonde with ground GPS data for electron density estimation

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Abstract

Dual frequency Global Positioning System (GPS) receivers provide integrated total electron content (TEC) along the ray path (slant TEC, affected by a bias). By inverting this observable, it is possible to obtain the vertical total electron content with some assumptions about the horizontal structure of the ionosphere.

The large number of permanent receivers distributed around the world provide enough information to obtain such TEC observables with high spatial and temporal resolutions. Nevertheless, the geometry (mainly vertical) of the ground GPS observations does not allow to solve the vertical structure of electron density of the ionosphere.

Mixing different kinds of complementary data in a tomographic context helps to overcome this problem. Several works have obtained successful results achieved by combining occultation and ground GPS data to estimate the local three-dimensional structure of ionospheric electron density. This paper proposes the use of just ground data to obtain similar or better results. To do this, the ground GPS data are mixed with vertical profiles of electron density derived from ionosonde data instead of GPS occultation observations.

In this paper, the complementarity between vertical profiles of electron density (estimated using the NeQuick model) and ground GPS data (from GPS IGS permanent network) are shown as well as the performance of the resulting combination.

Keywords: GPS; Ionosonde; GPSMET; Occultation; Electron density 3D description

1. Introduction

The main advantages of the ground Global Positioning System (GPS) data for ionospheric sounding is that one can expect to obtain accurate total electron content (TEC) estimates (with typical errors of few TECU). Moreover, for the time being, there are extensive GPS networks that provide the user with worldwide data availability. Nevertheless, this kind of data are unable to offer good vertical resolution for ionospheric tomography. On the other hand, ground-based ionosonde data offer high resolution up to the hmF2, but no real data are given about the topside ionosphere. Besides, only local vertical profiles are available.

Recognizing that the advantages of one data type may compensate for the weakness of the other, one can expect two main features of this method: the acquisition of information above hmF2 (i.e. the "topside"), and a certain degree of vertical resolution where ionosonde data is not available.

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Several efforts have been made in the field of data combination for ionospheric sounding using ground and LEO GPS receivers (see for example Hernández-Pajares et al., 1998). Following the idea of data combination, the present paper is focused on obtaining a 3D description of the ionospheric electron density distribution by combining ground GPS data and vertical profiles of electron density retrieved from ionosonde instead of GPS occultation data. To do this, a voxel approach has been chosen since it offers an easy and versatile way to combine data types with different features and properties (note that ionosonde data are only up to NmF2, whereas GPS data are up to 20 000 km height).

The paper is organized as follows: Section 2 (*Combining complementary data*) contains an explanation on how the method is carried out. Section 3 (*Results*) is divided into two scenarios: the first one consists of picking up the minimum number of vertical profiles from ionosondes (one or two at most) and expanding in longitude/latitude (using ground GPS data) their vertical resolution. To evaluate the performance of the method in this scenario, the ionosondes that have not taken part in the process have been used for comparison. The second scenario deals with the reconstruction of STEC profiles obtained by GPS/MET observations (using the proposed combination scheme) and offers a comparison with the actual ones. Finally, a section with the *Conclusions* ends the paper.

2. Combining complementary data

The ionospheric tomography considered in this work is based on voxels in which the electron density, in a Sun-fixed reference frame, is assumed to be constant within each cell. Previous works (Hajj et al., 1994) that have studied the validity of this approach state that the voxel size must be small enough so that the actual variations within a cell are low with respect to that constant value. As done in Hernández-Pajares et al. (1999), the size of these cells has been set to $7.5^\circ \times 5^\circ$ in local time and latitude, respectively. Taking into account that the data set considered occurs in the year 1995, during low solar activity, and the area of study is mid-latitude, this size does not introduce a significant mismodelling. Nevertheless, cell size should be reduced when studying low-latitude regions or periods with high solar activity.

To account for the vertical variability of the electron density, the ionosphere is divided in height as well. The configuration of the layer height is chosen based on geometric considerations, to be precise on how this configuration allows to distinguish between layers. This implies that cells must be large enough in height. The number of layers (eight in this work) is limited firstly by the geometry of data, and secondly by computer load.

The second step in this approach is to estimate the electron density using the carrier phase ionospheric combination of ground GPS receivers. In this work, we have used the $L_I \equiv L_1 - L_2$, i.e. the ionospheric (geometric free) combination of GPS carrier phases. This is due to the small error of this observable (millimeter level) compared to the error that offers $P_I \equiv P_2 - P_1$ or ionospheric (geometric free) combination of GPS code (meter level). The L_I in a voxel model can be written according the following equation (Hernández-Pajares et al., 1999, 2000):

$$L_I = k \cdot \sum_i \sum_j \sum_k (N_e)_{i,j,k} \cdot l_{i,j,k} + bias,$$

where L_I is expressed in length units, $(N_e)_{i,j,k}$ is the electron density in cell (i,j,k), $l_{i,j,k}$ is the length of the portion of ray in cell (i,j,k), the constant $k = 1.0506$ [meters of delay $m^2/10^{17}$ electron], and *bias* includes the unknown corresponding to the instrumental delays and phase ambiguity. The inverse problem is solved in order to find the electron densities and biases by means of a weighted least mean square approach. The reference frame considered is a Sun-fixed one, and the area of study is a regional (European) zone. Due to the Earth's rotation, it gives the system dynamics, the ground GPS and ionosonde stations illuminate cells corresponding to the selected European zone within a time span of less than 3 or 4 h. Within this period the electron density is considered stationary. Therefore, Kalman filtering is not necessary in this case. Nevertheless, this assumption may introduce a significant mismodelling during ionospheric storms.

The geometry of rays between GPS satellite and ground receiver is mainly vertical. This fact causes high correlations on the estimations of the different cells in the vertical because there is no reliable information about vertical distribution of the electron densities. In order to compensate for this lack, the combination of complementary data is considered. This combination is done in the last step of the process, where the estimation computed with GPS ground data is modified using constraints obtained with the additional data.

2.1. Complementarity of ionosonde data

The feasibility of tomography of the ionosphere has been conducted previously by several works (see for example Howe et al., 1998). In Hajj et al. (1994), the complementarity between ground and low Earth orbiter data was stated using simulated data, while on the other hand, Hernández-Pajares et al. (1998) performed ionospheric tomography using ground and space GPS real data. The reconstructed vertical profiles were compared with the actual ionosonde derived values.

Fig. 1 is an example in which the complementarity of models can be seen based on ionosonde data with respect to ground GPS data. In this figure, an example of an estimated profile (for Slough, see Fig. 3) is plotted using only ground GPS data and the solution after constraining with an ionosonde (in this case El Arenosillo is the constraining ionosonde). Before using vertical information (ionosonde), one can expect good estimations of TEC (GPS ground data solution). But regarding the vertical performance, since the

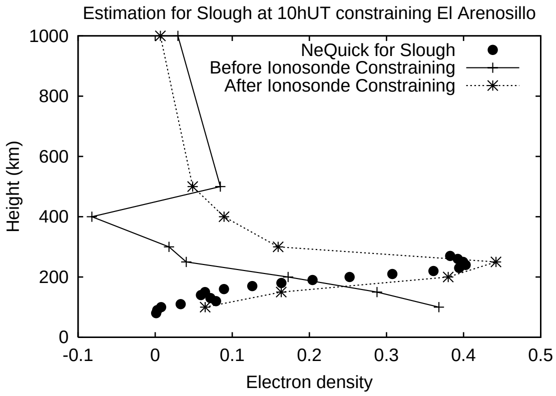

Fig. 1. Effect of including ionosonde data to GPS electron density estimations. In crosses the electron density in the voxels have been estimated using only ground GPS data and this nonrealistic profile gives an account of the high correlations when this data are used alone. The solution with asterisks is obtained after constraining the "ground GPS alone" solution with ionosonde (in this case only the data from *El Arenosillo* ionosonde have been constrained). Circles are the reference values corresponding to Slough ionosonde. The epoch for this plot is October 18th, 1995 10 hUT. Locations of GPS and ionosonde stations are depicted in Fig. 3.

values of cells in the vertical are highly correlated, the process gives nonrealistic profiles. If data with vertical information is introduced, these correlations are diminished giving a more realistic solution and results close to actual profiles derived from ionosonde data. This result shows that profiles of electron density modelled by using ionosonde data significantly helps to orientate the solution computed with only ground GPS data.

In this work, the vertical information of electron density will be given by the vertical profiles of NeQuick model instead of LEO GPS data. This model is constructed up to the hmF2 from ionosonde NmF2 and M3000 measurements, see Hochegger et al. (2000). Before constraining the GPS solution, the values of electron densities obtained from ionosondes are averaged in height (according to the width of the tomographic model layers). By doing this, one will have averaged the values of electron densities at the same heights that were selected to compute the GPS solution. After this has been done, one proceeds to apply the constraining scheme. This is divided into two parts:

(1) Up to hmF2, actual values of averaged electron densities from the constrained ionosondes are used to constrain the GPS solution. The reason is because the values of electron density under the hmF2 are calculated from observed parameters.
(2) Above hmF2, ratios between values of modelled electron density are used to fix a relationship in height between densities in cells. This type of constraint is used to consider a model for the topside ionosphere (the NeQuick values for the topside are based on a semi-Epstein function).

To compute the final solution a Gauss–Markoff model with constraints (see Koch, 1988) obtained from ionosondes has been implemented, which is equivalent to stating that the data coming from ionosonde are given a very high weight than the GPS data.

3. Results

The formal errors will depend, among other factors, on the noise level of the measurement and the relative weights assigned to each of the data types considered in the procedure (in this work an error of $0.1\,TECU$ for all GPS phase measurements has been assumed). To determine the value of these weights may be a difficult task and, besides, the formal error of the model may be realistic only to a limited extent and only useful to discard the estimates with large errors (outliers). The formal errors obtained show typical values of $0.1e10e/\mathrm{m}^3$. These values provide an RMS $\approx 1\,TECU$ in the observation minus calculus (residual).

To complete the evaluation of the error, actual measurements that have not taken part in the estimation of the electron densities have been used for comparative purposes. In this context, and based on the different real data considered to evaluate the performance, two different scenarios are proposed: (1) estimation of electron density using data from ground GPS receivers and the minimum number (one or two at most) of ionosondes, and (2) reconstruction of GPS/MET occultations STEC using data from ground GPS receivers and all ionosonde data available. This study has been carried out for the days of October 17th, 18th and 19th, 1995. An ionospheric storm began in the evening of day 18 as shown in Fig. 2. The GPS network used in this work and the ionosonde locations correspond to the European region (see Fig. 3). The distances between ionosondes can be seen in Table 1. The centers of the eight considered layers have been placed at 100, 150, 200, 250, 300, 400, 500 and 1000 km. The boundaries of the layers can be computed as the mid-distance between centers and it has been considered an ionosphere is considered to start at 60 km and end at 2000 km.

3.1. Vertical profile reconstruction

The experiment of this section consists of computing vertical profiles using the ground GPS network and profiles up to hmF2 derived from the minimum number (one or two at most) of ionosonde measurements using NeQuick model. To evaluate its performance, the resulting profiles have been compared with the ones provided by the remaining ionosondes that have not taken part in the process. Note that the vertical resolution obtained with the proposed method is limited to eight shells, thus the comparison is done with the resulting averaged value of electron density in the same layers

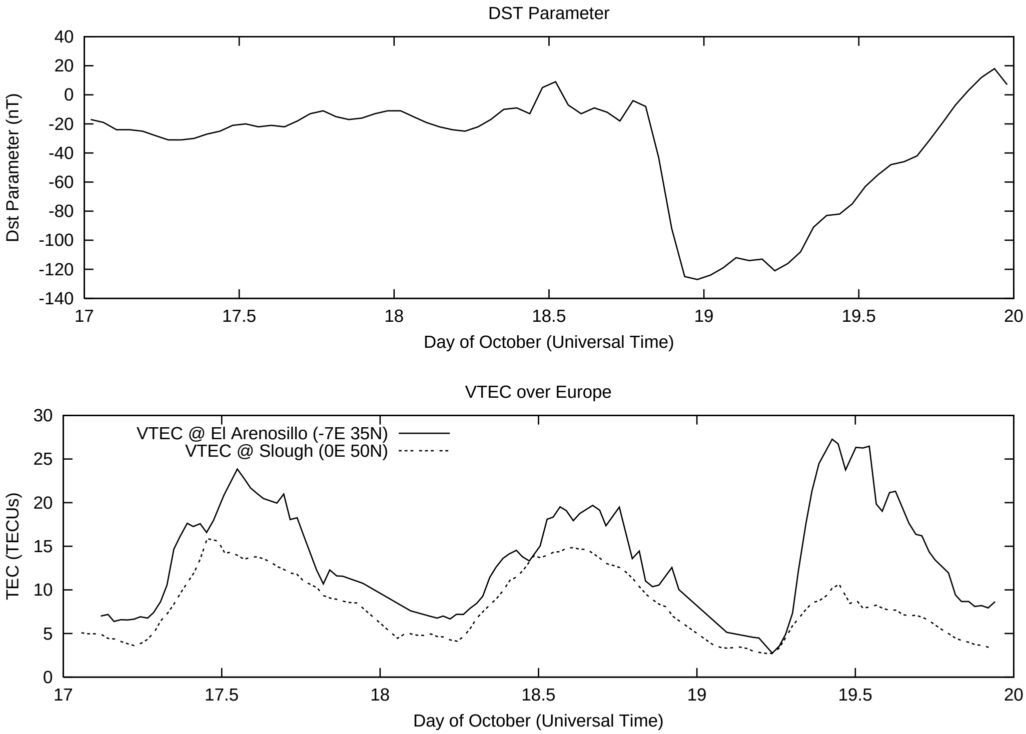

Fig. 2. Evolution of the DST parameter and VTEC during the study days of October 1995, 17th, 18th and 19th.

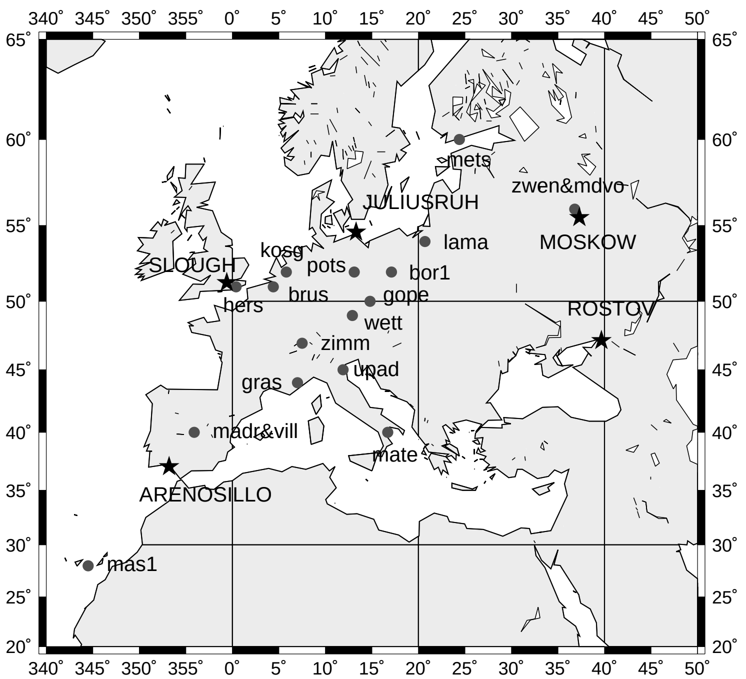

Fig. 3. Figure depicting GPS network and ionosonde stations where vertical profiles were obtained. Circles correspond to IGS GPS stations and stars to ionosonde stations.

Table 1
Table of approximate distances in km between ionosondes

	Arenosillo	*Juliusruh*	*Moskow*	*Rostov*
Slough	1700	1600	4200	4500
Rostov	5300	3000	1000	
Moskow	5300	2700		
Juliusruh	3000			

computed with NeQuick vertical profiles (vertical resolution of 10 km). Fig. 4 shows two examples of performance with the corresponding RMS, computed as the difference between values of electron densities at different heights.

Table 2 shows the RMS of the comparison between the computed and NeQuick profiles in several cases, as well as the mean difference of hmF2 and NmF2. In the hmF2 comparison takes into account that the lowermost cells are more or less 50 km in width and this point introduces an additional source of error (the resolution of the method is 25 km in hmF2). Moreover, since the values of electron densities are averaged in height as explained in Section 2.1, this introduces an additional source of error in the NmF2 comparison. The table has been divided in order to show the performance in low and high geomagnetic activity conditions, DST value above or under −40 nT, respectively (it has only been considered the beginning of the ionospheric storm, before October 20th, which does not include the complete evolution of it) and with one and two ionosondes as constraints. Initially, only *El Arenosillo* is constrained. The choice of this ionosonde is to diminish the spatial–temporal correlation between the constrained and test ionosondes. In this case, the error on high geomagnetic activity is clearly greater than the low activity case. This is due to the fact that the storm affected the central and northern ionosondes in a different way (slight decrease of VTEC over these locations) with respect to those located in southern Europe (see Fig. 2). Therefore, since *El Arenosillo* ionosonde was selected to estimate the northernmost ionosondes, the results worsened significantly. To lessen this error, it was necessary to include to the model data from an ionosonde that was equally affected by this storm. Note that when a second northern ionosonde is added, the effect of the storm in the estimation error decreases.

The lack of 100% agreement in the estimation of the constrained ionosondes data is due to the interpolation scheme used in this work. The grid used is not adapted to the ionosonde coordinates. Thus, to obtain the estimation of a profile, an interpolation between the four nearest grid centers is carried out. This explains why the error in the estimation of *Juliusruh* is smaller than the one from *El Arenosillo* when both are constrained: the coordinates of the former happen to be close to one of the interpolated grid centers, thus the value of these grid centers are directly the constraining ones and the remaining grid centers' contributions (and thus their errors) are small. In the case of the *El Arenosillo*, the contributions of the four grid centers are equally important, thus diminishing the effect on the constraint over this station. From Table 2, it can be seen that this interpolation error ranges between $4e10e/\mathrm{m}^3$ (worst case, *El Arenosillo*) and $1e10e/\mathrm{m}^3$ (*Juliusruh*). The possibility of adapting the grid centers to the ionosonde coordinates improves the estimations of the constrained ionosondes. But there is no clear improvement in the estimated ionosondes and, besides, this feature increases the needed computer memory and computing time. In high geomagnetic activity conditions the high variability of the ionosphere may increase this interpolation error.

A more graphical comparison can be seen in Fig. 5 where the 1 to 1 relation between the NeQuick values of NmF2 (x-axis) and the computed ones (y-axis) has been plotted. It can be seen that Slough, which is inside the area limited by the coordinates of Arenosillo and Juliusruh, has less estimation errors than Rostov.

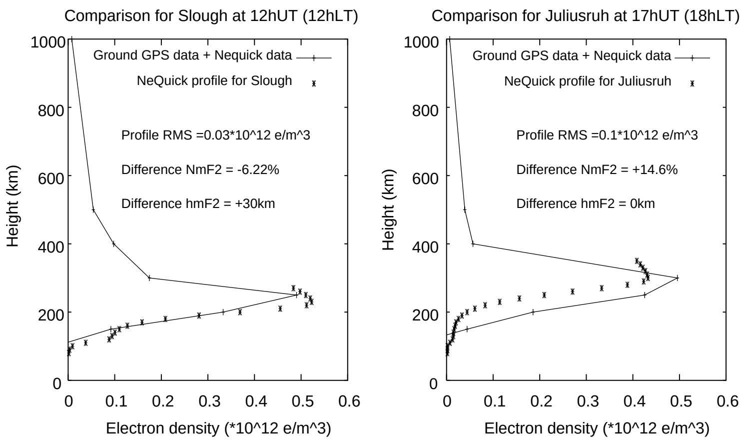

Fig. 4. Examples of performance and respective RMS for two different stations at different times when only Arenosillo ionosonde has been constrained. These plots correspond to October 18th, 1995, previous to ionospheric storm (Dst above −40).

Table 2
Comparison between the computed electron density profiles with respect to those provided by the NeQuick model derived from ionosonde measurements

	Low geomagnetic activity		High geomagnetic activity	
	Profile RMS	Diff. NmF2/Diff. hmF2	Profile RMS	Diff. NmF2/Diff. hmF2
*Arenosillo**	4 (112)	−15.5%/2.1 km (24)	6 (107)	−22.7%/ − 16.3 km (19)
Juliusruh	6 (104)	−18.6%/ − 3.7 km (19)	19 (85)	139.2%/ − 54.6 km (13)
Moskow	8 (116)	−6.4%/ − 17.1 km (21)	15 (70)	−9.1%/ − 166.2 km (8)
Rostov	7 (68)	−23.8%/ − 25.0 km (12)	16 (56)	77.1%/ − 66.7 km (6)
Slough	5 (88)	−4.7%/16.9 km (13)	25 (42)	103.3%/ − 87.5 km (8)
*Arenosillo**	4 (111)	−11.2%/2.1 km (24)	6 (107)	−27.7%/ − 16.3 km (19)
*Juliusruh**	1 (97)	−10.0%/ − 5.0 km (18)	2 (84)	−25.6%/ − 31.5 km (13)
Moskow	6 (102)	15.8%/ − 15.6 km (18)	5 (67)	−54.0%/ − 81.9 km (8)
Rostov	5 (56)	−5.4%/ − 31.1 km (9)	9 (56)	−29.0%/ − 83.3 km (6)
Slough	4 (84)	−0.3%/16.9 km (13)	5 (42)	5.7%/ − 31.2 km (8)

*The table gives the RMS (divided by $10^{10} e/m^3$) of the difference between the complete profiles up to the maximum and the averaged difference of NmF2 and hmF2. It presents different situations, for low and high geomagnetic and with different ionosondes constrained (marked with an asterisk). In parentheses are the number of comparisons made for each case (all points of the profile for the RMS comparison and only the maximum for the NmF2 and hmF2 comparison).

3.2. GPS/MET occultations reconstruction

From April 1995–1997, the GPS/MET low Earth orbiter gathered, at 740 km approximately, the dual-frequency codes and phases of GPS satellites offering a valuable source of information for atmospheric sounding and in particular for ionospheric tomography. Works like Hajj et al. (1994) or Leitinger et al. (1997) used the ionospheric delay (proportional to the slant TEC) measured by GPS/MET to provide tomographic models with vertical information of electron density distribution. In this section, this observable will be used for validation purposes of the proposed approach.

In this scenario, densities using ground GPS and all available ionosondes have been computed. Afterwards, each ray from each GPS satellite to GPS/MET has been divided according to the cells it crosses. Once this has been done, a reconstructed STEC is generated by means of multiplying the length of each portion of the ray by the corresponding electron density. Once all occultations have been reconstructed, they are compared with actual GPS/MET measurements.

As is known, an occultation consists of a group of consecutive rays seen with negative elevations. Fig. 6 shows a single ray of an occultation (upper) and the corresponding projection on a 2-D map of all rays that form an occultation (lower) using the same color coding. In the lower map of Fig. 6, the range of latitudes/longitudes covered by a complete occultation can be seen as well. The study of the occultations provided by GPS/MET reflects that a great deal of rays go through the height where there is a maximum of electron density, thus providing valuable data to test the performance of its estimation in these regions.

In Fig. 7, an example of reconstruction is given, the STEC (in TECU) is plotted against the minimum distance between

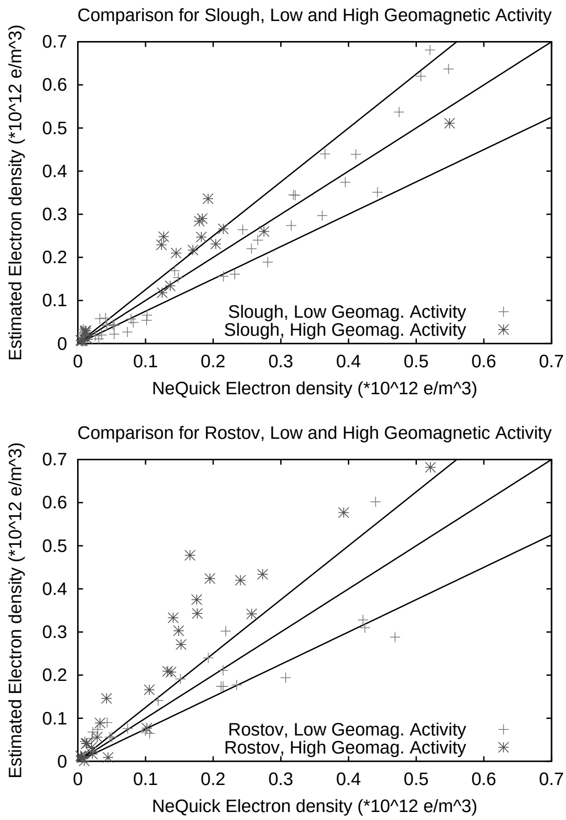

Fig. 5. Comparison of estimations near hmF2 when Arenosillo and Juliusruh profiles have been constrained. The middle line corresponds to 100% agreement while the side lines corresponds to agreement with 25% error.

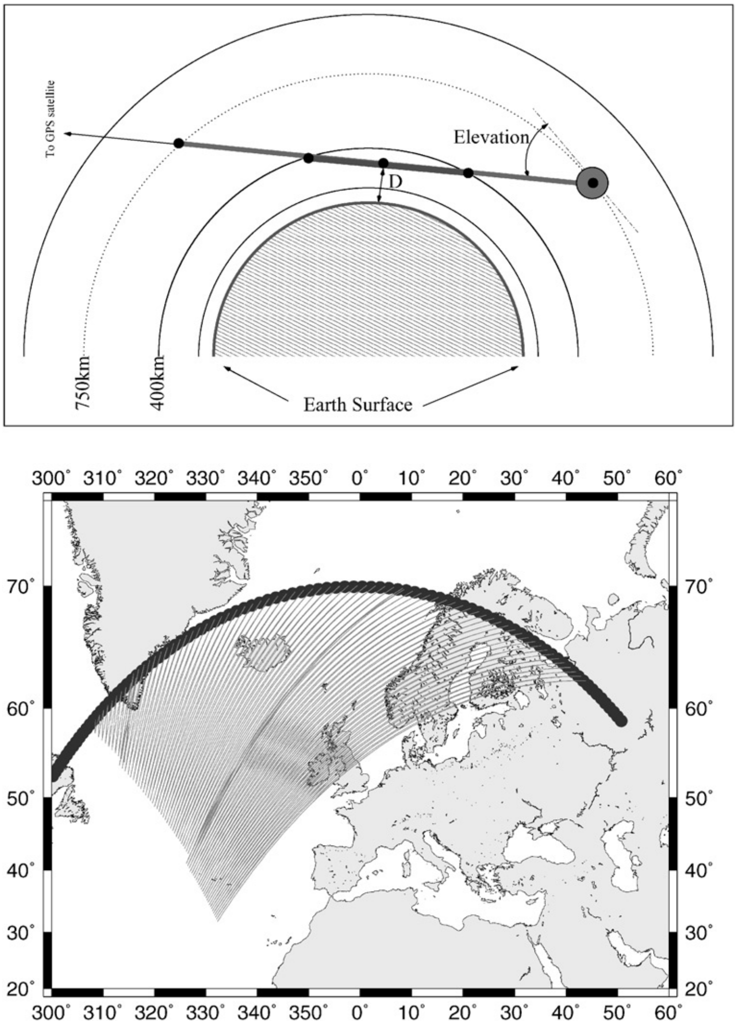

Fig. 6. Occultation of GPS/MET. Light grey lines in this figure reveal the portion of the ray comprised under the GPS/MET orbit height. On the other hand the dark grey lines indicate the portion of ray that crosses the region of maximum of electron density (under 400 km). Black line depicts the orbit of GPS/MET.

ray and earth surface (impact parameter D, see Fig. 6). The peak corresponds to the rays affected by the NmF2.

To give a more general impression in Fig. 8 the modelled STEC against the observed STEC for all occultations considered on October 18th is shown (30 occultations, 1641 points to compare). It can be seen that the agreement is better for the reconstructions under the hmF2. This is due to the fact that under this height there are more resolutions in height (check the configuration of the height layers centers) and the constraints are built from a model anchored with ionosonde measurements (better constraint), leading to better estimations of electron density. This point can be seen in Fig. 7 as well, where for impact parameters above the maximum there is a coarseness in the estimation. This is due to the fact that the upper layers are large in height and this implies worse resolution in the estimations.

3.2.1. *Comparison with Abel Inversion*

The observations gathered by GPS/MET have led to the use of inversion techniques such as *Abel Inversion* (Hajj and Romans, 1998; Schreiner et al., 1999) to perform vertical profiling of the ionosphere. Hernández-Pajares et al. (2000) showed that it is possible to retrieve profiles with Abel Inversion from GPS occultations with a confidence of 10% on foF_2 and FoE with respect to raw ionosonde data. This fact suggests that an equivalent way to check the results obtained in this paper is to compare the vertical profiles obtained with Abel inversion techniques with those computed with the proposed method. Note that the results are obtained with different data, that is: Abel Inversion uses only GPS/MET data while the proposed method uses ground ionosonde and ground GPS receivers. Although both methods are independent, they give similar results (see Fig. 9 for an example of

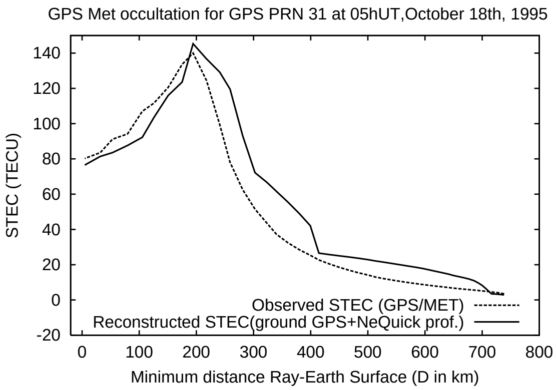

Fig. 7. Example of single occultation modelled with the ground GPS data and the NeQuick vertical profiles compared with the observation of GPS/MET.

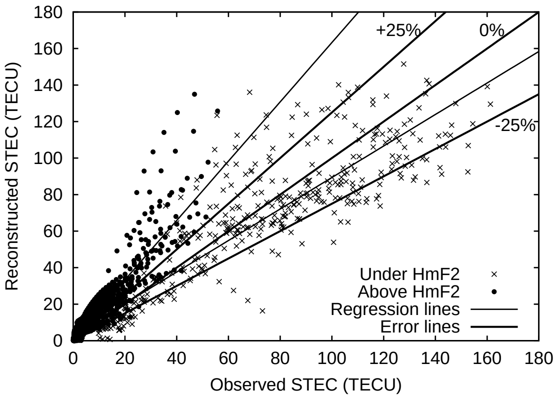

Fig. 8. Comparison between observed and modelled STEC. Constrained by all ionosondes, October 18th, 1995. Middle line corresponds to perfect agreement while the side lines correspond to a 25% of error. The correlation coefficients for linear regression above and under hmF2 are 0.819 and 0.755, respectively.

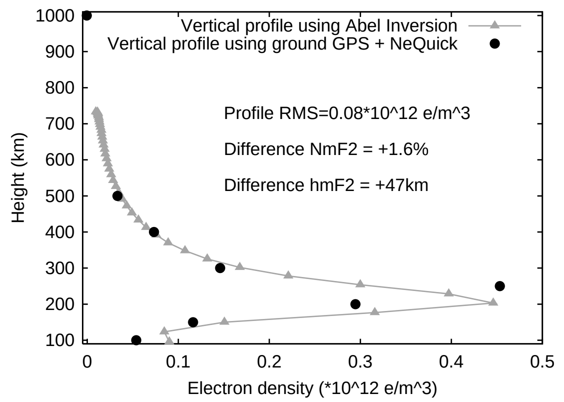

Fig. 9. Comparison between profiles obtained with Improved Abel Inversion (see Hernández-Pajares et al., 2000) and those estimated with ionosonde and ground GPS data.

comparison between both techniques). This shows how the technique explained in this work can give relevant information about topside ionosphere.

4. Conclusions

In this work, the feasibility of data combination has been shown between GPS ground data and vertical profiles computed from ionosonde data. The performance has been studied in two different scenarios:

(1) Electron density vertical profiles have been reconstructed using only one ionosonde and ground GPS data.
(2) STEC profiles seen by GPS Met occultations have been reconstructed using all ionosonde and ground GPS data.

The results obtained suggest that it is possible to reconstruct the electron density in regions where ionosonde data are not available by combining ground GPS and relatively far ionosonde data. The differences in this context are in general better than 25% in low geomagnetic activity conditions. Moreover, valuable information about the topside can be obtained using constraints based on ratios extracted from a certain model. Regarding the bottomside, it can be estimated using realistic data from a model like NeQuick, combined with measured ionosonde data.

The estimation of GPS/MET revealed that it is possible to obtain reconstructions of occultations with errors less than 25% for estimations under hmF2. To improve the accuracies obtained above this point more realistic plasmaspheric models may be included. Once this topic has been studied, the validation scheme may consist of reconstructing non-occultations observations (with positive elevations) of GPS/MET or other LEO satellites.

Future improvements of this model may be considered. The interpolation scheme between grid centers may be improved in order to diminish the interpolation error. Moreover, new types of data such as those obtained from topside electron density information provided by ionospheric sounders onboard satellites, 3D ionospheric models like NeQuick in its full version or UV information may be considered for further improvements on data combination. Hajj and Romans, 1998

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