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Application of ionospheric tomography to real-time GPS carrier-phase ambiguities resolution, at scales of 400-1000 km and with high geomagnetic activity

M. Hernández-Pajares, J. M. Juan and J. Sanz

Group of Astronomy and Geomatics, Universitat Politècnica de Catalunya, Barcelona, Spain

O. L. Colombo

USRA/NASA Goddard SFC, Code 926, Maryland, USA

Abstract. The influence of the ionosphere can be one of the main obstacles to GPS carrier phase ambiguity resolution in real-time, particularly over long baselines. This is important to all users of GPS requiring sub-decimeter positioning, perhaps in real time, especially with high geomagnetic activity or close to the Solar Maximum. Therefore, it is desirable to have a precise estimation of the ionospheric delay in real-time, to correct the data. In this paper we asses a real-time tomographic model of the ionosphere created using dual-frequency phase data simultaneously collected with the receivers of a network of stations in the USA and Canada, with separations of 400-1000 km, during a period of high geomagnetic activity (Kp=6). When the tomographic ionospheric correction is included, the resolution on-the-fly (OTF) of the widelane double-differenced ambiguities at the reference stations is nearly 100% successful for satellite elevations above 20 degrees, while the resolution of the L_1, L_2 ambiguities at the rover is typically more than 80% successful.

Introduction

By affecting its signals, the distribution of free electrons in the ionosphere —very dependent on the solar cycle and the "space weather"— impedes the use of Global Positioning System (GPS) for precise navigation. The most precise form of GPS positioning is relative positioning based on the carrier phase. The observations from the rover receiver and from fixed receivers forming a network of reference (or control, or fiducial) stations, are processed simultaneously, to eliminate common sources of error. The resulting positions are relative to those of the stations which, in turn, must be precisely known. It is common practice to use as data either carrier phase or pseudo-range double differences, formed by subtracting observations of pairs of satellites made simultaneously with pairs of receivers. How the ionospheric effect on the data may be handled depends on the size of the area covered by the reference stations. Accordingly, differential GPS positioning can be classified as: (1) Local Area Differential GPS (LADGPS), when the rover receiver is less than few tens of km from any reference GPS station (see for example *Pratt et al.* 1998). The assumption that the ionospheric delays are identical for both stations and can be ignored in order to fix the double-differenced integer ambiguities in L_1 and L_2, strongly depends on the geomagnetic activity and possible existence of ionospheric perturbations such as Travelling Ionospheric Disturbances (TID) (*Coster et al.* 1998). (2) Regional Area Differential GPS (RADGPS), when the dual-frequency rover receiver is at distances of few hundred km from the reference stations. *Gao et al.* (1997) showed an example in which they succeeded in fixing the ambiguities of double-differenced carrier phase by indirectly estimating the ionospheric delay by an integer search procedure, without an ionospheric model (this was done close to minimum Solar Cycle conditions). (3) Wide Area Differential GPS (WADGPS), when the rover receiver may be 200-500 km from the nearest reference receiver. The best known approach is to use the carrier-phase smoothed L1 pseudo-range to get real-time positioning errors at the meter level, when modeling the ionospheric delay, and other undesirable effects, such as tropospheric delay (*Enge et al.* 1996).

The dual-frequency GPS signals, gathered from a set of reference receivers, can be used to compute a real-time ionospheric model in the RADGPS and WADGPS cases, to provide a precise ionospheric correction to the GPS navigator. The purpose of this paper is to assess, for a large network and higher geomagnetic activity than in *Colombo et al.* 1999, how well the ionospheric tomography can help resolve carrier phase ambiguities, so as to achieve very precise positioning in real-time.

The real-time tomographic model

The free electron density can be described as a random walk process in time that can best be estimated in a Sun-fixed reference frame where it is relatively stationary (variation of ±10% during one day in mean latitudes and Solar Maximum conditions). The tomographic model adopted is spatially formed by a set of cells or volume elements (voxels), especially suitable to detect local features, that cover all the ionosphere sampled by the GPS satellite/receiver rays. These voxels, which electron density is considered uniform at any given time, can be taken with the same size for describing a region sampled from an approximately homogeneously distributed network of reference stations. A voxel size of 3x5 degrees in latitude and solar longitude, and two layers with boundaries at 60-740-1420 km have been adopted. This is adequate to get precise ionospheric determinations from ground GPS data (Hernández-Pajares et al. 1999).

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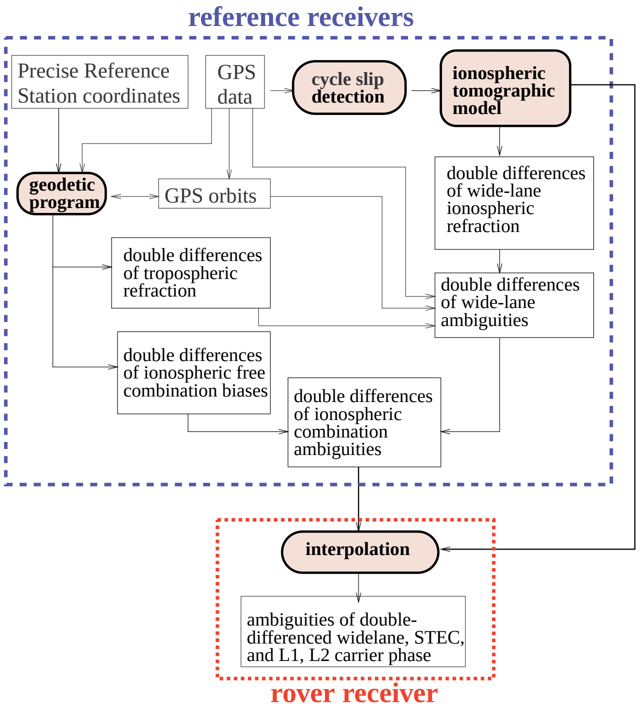

Figure 1. Flow chart summarizing the new strategy.

The resolution of the model, by means of the Kalman filter, initialized with data from the previous day, is performed using the geometry-free combination of phases, L_I, of the transmitter T measured from the receiver R:

$$L_I = L_1 - L_2 = \alpha \sum_i \sum_j \sum_k (N_e)_{i,j,k} \Delta s_{i,j,k} + b \qquad (1)$$

being L_1 and L_2 the carrier phases (in meters) at frequencies $f_1 = 154 f_0$ and $f_2 = 120 f_0$ ($f_0 = 10.23$ MHz); α=1.05 meters of L1-L2 / $10^{17} \frac{e}{m^2}$; where i, j, k are the indices for each cell corresponding to solar longitude, geodetic latitude and height; $(N_e)_{i,j,k}$ is the free electron density; and $\Delta s_{i,j,k}$ is the length of the ray path crossing the "illuminated cells"; and b is the alignment term (constant in a given transmitter-receiver arch of continuous phase) that includes the L_1, L_2 integer ambiguities and instrumental delays. This approach extends the model described in Hernández-Pajares et al. (1998) to real-time applications. The estimation of this ionospheric model is done by means of a Kalman filter with 10 minutes of updating time (similar performance with 2 minutes), in such a way that the results of the last batch are used to estimate the ionospheric delays up to the next updating time. Then, all ionospheric delays are estimated only from the previous data, as must be done in real-time.

Fixing the double-differenced carrier phase integer ambiguities

In *Colombo et al.* 1999, the resolution of the carrier phase ambiguities is studied for a network of 200-300 km separation between sites. This study shows the value of using an accurate tomographic ionospheric model to solve in real-time the ambiguities in the widelane combination first, and then in L_1 and L_2, for the reference stations (and also for the rover, both a fixed site and a moving car).

Indeed, if we consider the widelane combination for the reference stations $L_\delta = (f_1 L_1 - f_2 L_2)/(f_1 - f_2)$, then its double difference (indicating the difference satellite to reference satellite with (∇), and station to reference station with (Δ)) can be written as

$$\nabla\Delta L_\delta = \nabla\Delta\rho + \nabla\Delta T + \nabla\Delta I_\delta + \lambda_\delta \nabla\Delta N_\delta \qquad (2)$$

ρ being the distance satellite-receiver, T the tropospheric delay, I_δ the widelane ionospheric correction and N_δ the integer widelane ambiguity (centimetric terms like the phase multipath have been neglected).

To fix in real-time $\nabla\Delta N_\delta$ to the right integer value no less than 95% of the time, it is necessary to calculate the other three terms in equation 2 with a maximum total error of less than $\lambda_\delta/2 \simeq 40$ cm, i.e. with an error standard deviation of less than $\simeq 20$ cm. An error of few centimeters can be expected for the satellite-receiver distance term $\nabla\Delta\rho$, if the satellite positions are obtained from extrapolated precise ephemeris or from real-time corrected broadcast ephemeris (*Colombo and Evans,* 1998). Regarding the double-differenced tropospheric correction $\nabla\Delta T$, and for stations at distances of few tens of km, the maximum error using the models for the hydrostatic and wet components is typically lower than 10 cm for elevations greater than 20 degrees (*Coster et al.* 1998, Figure 5d). But this error could diminish potentially to a few centimeters if the usually slow-varying tropospheric refraction is estimated in real-time, in particular in the reference stations where the coordinates can be accurately known (see flow chart in Figure 1). In our case, using precise orbits and modeled tropospheric corrections, a final maximum error of $\simeq 30 - 40$ *cm* in $\nabla\Delta I_\delta$ is allowed, considering the error budget of the terms in equation 2. This means a standard

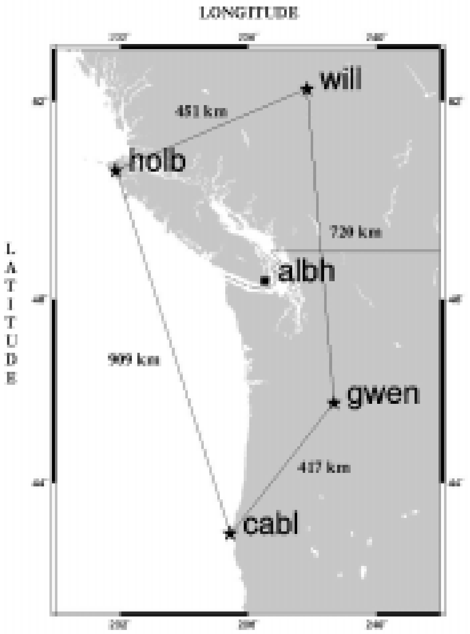

Figure 2. Network of stations used for the study (stars = reference stations, square = test station). HOLB was the reference site for forming double differences and positioning the test station (map generated with GMT).

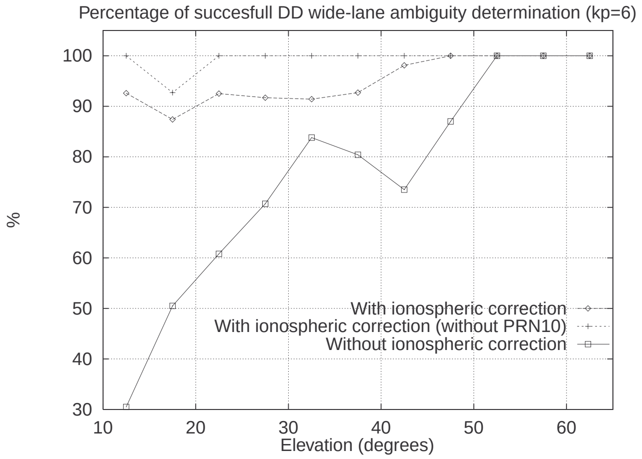

Figure 3. Percentage of successful double-differenced widelane integer ambiguity determination as a function of the elevation of the lowest satellite, for high geomagnetic activity (Kp=6).

deviation of ≤ 20 cm for the ionospheric correction of the widelane combination to guarantee the 95% successful determination of $\nabla\Delta N_\delta$ (i.e. an electron content standard deviation of 1 TECU=10^{16}electrons/m^2, $\simeq 10$ cm in L_I, $\simeq 15$ cm in L_1, $\simeq 20$ cm in L_δ).

Once $\nabla\Delta N_\delta$ is fixed, it is possible to fix the L_1 and L_2 double-differenced integer ambiguities N_1 and N_2 for the reference stations, using a sufficiently accurate determination of the double-differenced ambiguity $\nabla\Delta B_c$ of the ionospheric free combination $L_c = (f_1^2 L_1 - f_2^2 L_2)/(f_1^2 - f_2^2)$. The following relationships illustrate these steps:

$$\begin{aligned}
&\nabla\Delta B_c = 0.5[\lambda_\delta \nabla\Delta N_\delta + \lambda_n \nabla\Delta(N_1+N_2)]\\
&\nabla\Delta(N_1+N_2) = \mathrm{NI}[(2\nabla\Delta B_c - \lambda_\delta \nabla\Delta N_\delta)/\lambda_\nu]\\
&\nabla\Delta N_1 = 0.5[\nabla\Delta N_\delta + \nabla\Delta(N_1+N_2)]\\
&\nabla\Delta N_2 = \nabla\Delta N_1 - \nabla\Delta N_\delta
\end{aligned} \tag{3}$$

being $\lambda_n = c/(f_1+f_2) \simeq 10.7$cm and NI the nearest integer. Hence, from $\nabla\Delta N_1$ and $\nabla\Delta N_2$, the unambiguous double-differenced ionospheric slant total electron content, STEC, can be computed for the reference stations:

$$\alpha \nabla\Delta STEC = \nabla\Delta(L_1 - L_2) - (\lambda_1 \nabla\Delta N_1 - \lambda_2 \nabla\Delta N_2) \tag{4}$$

Finally, and following the procedure described in *Colombo et al.* 1999 (see also the flow chart in Figure 1), it is possible to solve the carrier phase ambiguities for a roving receiver by interpolating the values of the unambiguous double-differenced ionospheric STEC from the reference stations to the rover. Indeed, assuming that the $\nabla\Delta B_c$ bias in the rover is estimated simultaneously with its position, we can find from $\nabla\Delta L_\delta - \nabla\Delta L_c$ the widelane ambiguity if the error in the estimation of $\nabla\Delta B_c$ is less than $\frac{1}{4}$ of the widelane wavelength. In addition, if the error of the interpolated STEC is below $\frac{1}{2}|\lambda_1 - \lambda_2| \simeq 2.7$cm, we can solve for all the remaining entire ambiguities, as it can be deduced from the the following relationship:

$$\nabla\Delta(L_1 - L_2) = \alpha \nabla\Delta STEC + \nabla\Delta N_\delta \lambda_1 + \nabla\Delta N_2 (\lambda_1 - \lambda_2) \tag{5}$$

In this work we have applied the linear interpolation of unambiguous $\nabla\Delta STEC$ proposed by *Gao et al.* 1997.

Results and conclusions

The data set (Kp$\simeq$6, May 3rd, 1998, 20-23 hours UT) corresponds to four North-American reference stations belonging to the International GPS Service (IGS) network with distances of between 400 and 1000 km (CABL, GWEN, HOLB, WILL). The dual frequency GPS observations, with a sampling period of 30 seconds, are the inputs of the real-time ionospheric model, as it has been described in section 2. Also one test station (ALBH) is considered, whose actual double-differenced STEC will be compared to the model predictions (see map in Figure 2). All double differences were formed with HOLB as the reference site.

The computations have been performed simulating the real-time mode in the critical steps: The double-differenced troposphere, B_c and orbit errors should be estimated by a geodetic program, using the well-known positions of the reference receivers (see Figure 1). In this proof of concept, we have used: (1) Precise orbits instead of precise predicted orbits, and (2) standard hydrostatic tropospheric model (exponential dependence on height) without wet delay estimation.

The ionospheric double difference residuals have an RMS of 9 cm (0.9 TECU approximately) in L1-L2 delay units, which is below the critical value of 1 TECU discussed in the previous section, allowing to solve successfully most of the widelane ambiguities. Indeed, more than 90% of successful widelane integer ambiguities fixing is obtained for GPS satellite elevations above the horizon greater than 20 degrees (100% without the satellite PRN10 observed to the South), in contrast with the minimum elevation of 50 degrees when the ionospheric correction is neglected (Figure 3). The minimum elevation for solving successfully the double differenced widelane ambiguities with no ionospheric correction can be considered as an indicator of the ionospheric activity above the network. For instance, for the following day, May 4th, 1998, between 20-23 hours UT, with moderate geomagentic activity (Kp$\leq$3), the minimum elevation is 35 deg. And during the same day, 04-05UT with very severe ionospheric storm conditions (Kp=8.5) 100% is not achieved at any ele-

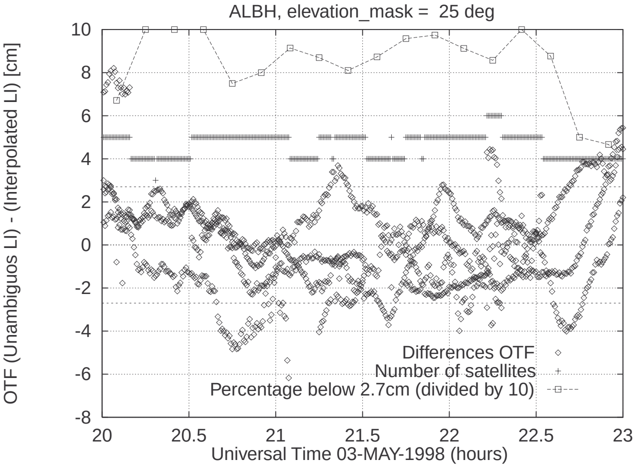

Figure 4. Percentage of successful determination of the unambiguous $\nabla\Delta STEC$ (better than 2.7 cm), for the test station ALBH and satellites with elevation greater than 25 degrees, as a function of the universal time (May 3rd 1998, Kp=6). Also the $\nabla\Delta STEC$ residuals and the number of satellites are indicated.

vation. When the ionospheric correction is applied, however, the results are similar for periods of both high and moderate activity. In the case of the severe storm, the model improves the widelane ambiguity estimation, with 60% to 90% success, in despite of the poor carrier phase quality.

As it has been commented above, by interpolating the unambiguous $\nabla\Delta STEC$ values at the reference stations, it is possible to solve the full set of ambiguities, when this value is more accurate than 2.7 cm. We attempted to resolve all simultaneously available double-differenced ambiguities on the baseline from HOLB to ALBH, which was not included in the set of reference stations. Figure 4 shows the percentage of those ambiguities properly resolved in each case, as a function of time and for periods of ten minutes. Most of the time, this happens with a success rate of 80-100%. The common reference satellite chosen when forming double differences (PRN30) is at a low elevation in the last part of the period, and this is coincident with the worst results between 22.5 and 23 hours UT.

We conclude that the tomographic model of the ionosphere is precise enough (better than 1 TECU in the double differences of the slant TEC) for the successful resolution, in real-time, of the widelane ambiguities. This holds true even with high geomagnetic activity (Kp=6), and distances up to 1000 km between reference stations. We observe than in this trial (with more and longer trials still to come), for an uninterrupted period of two hours, better than 80% of the STEC ambiguities at the test station have been resolved correctly.

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References

Colombo, O.L., and A.G. Evans, Testing decimeter-level, kinematic, differential GPS over great distances at sea and on land, ION GPS'98, Nashville, USA, September 1998.

Colombo, O.L., M. Hernández-Pajares, J.M. Juan, J. Sanz and J. Talaya, Resolving carrier-phase ambiguities on-the fly, at more than 100 km from nearest site, with the help of ionospheric tomography, ION GPS'99, Nashville, USA, September 1999.

Coster, A.J., M.M. Pratt, B.P. Burke and P.N. Misra, Characterization of atmospheric propagation errors for DGPS, ION 54th Annual Meeting, Denver, USA, June 1998.

Enge, P., T. Walter, S. Pullen, C. Kee, Y.-C. Chao and Y.-J. Tsai, Wide area augmentation of the global positioning system, Proceedings of the IEEE, vol. 84, no. 8, p. 1063-1088, 1996.

Gao, Y., Z. Li and J.F. McLellan, Carrier phase based regional area differential GPS for decimeter-level positioning and navigation, ION GPS'97, Kansas City, USA, September 1997.

Hernández-Pajares, M., J.M. Juan, J. Sanz and J.G. Solé, Global observation of the ionospheric electronic response to solar events using ground and LEO GPS data, Journal of Geophysical Research 103, A9, 20789-20796, 1998.

Hernández-Pajares, M., J.M. Juan and J. Sanz, New approaches in global ionospheric determination using ground GPS data, Journal of Atmospheric and Solar-Terrestrial Physics 61, 1237-1247, 1999.

Pratt, M., B. Burke and P. Misra, Single-epoch integer ambiguity resolution with GPS-GLONASS L1-L2 Data, ION GPS'98, Nashville, USA, September 1998.

M. Hernández-Pajares, J. M. Juan and J. Sanz, Group of Astronomy and Geomatics, Universitat Politècnica de Catalunya, Campus Nord Mod. C-3, Jordi Girona 1, E08034-Barcelona, Spain. (e-mail: manuel@mat.upc.es)

O. L. Colombo, USRA/NASA Goddard SFC, Code 926, Maryland, USA.

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