Resolving Carrier-Phase Ambiguities On The Fly, At More Than 100 km From Nearest Reference Site, With The Help Of Ionospheric Tomography

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ABSTRACT

In this paper we address the question of how to resolve GPS carrier phase ambiguities precisely and quickly, when the rover is more than 100 kilometers from the nearest reference site, in order to obtain sub-decimeter (r.m.s.) position in real-time (and, by implication, in post-processing). To do this reliably, ionospheric refraction has to be corrected very accurately. For distances of up to a few hundred kilometers, dual-frequency GPS data from permanent control stations may be used to obtain the necessary ionospheric refraction information. The ionosphere over the area served by the stations has to be mapped using their carrier phase data, also in real-time, by computed ionospheric tomography. The resulting map is used to produce information that is transmitted to the user, along with range and time corrections. The user can then calculate very precise corrections for ionospheric refraction at the location of the roving receiver. After a successful preliminary test using 1997 data from the core control stations of the CATNET network in Catalunya, (Catalonia) Spain, a second test involving those, as well as a road vehicle, was conducted in March of 1999, at a distance of 116 km from the nearest reference station. The

data were processed after the test, but simulating a real-time analysis. As reported here, repeated attempts at resolving all the L1 and L2 ambiguities proved successful over a total period of two hours.

INTRODUCTION

Using what is known as Wide Area Augmentation Systems (WAAS), Global Positioning System (GPS) pseudo-range data are used to position aircraft in real-time, with meter-level precision, relative to reference stations hundreds of kilometers away. If carrier phase measurements were used instead of pseudo-range, sub-decimeter precision could be obtained. However, phase measurements are ambiguous, and the exact resolution of their ambiguities is prevented by ionospheric refraction beyond a few tens of kilometers. This distance will decrease as ionospheric activity rises with the approaching Solar Maximum, expected to happen in early 2000. As figure 1 shows, solar activity is expected to remain quite high for several years after the maximum.

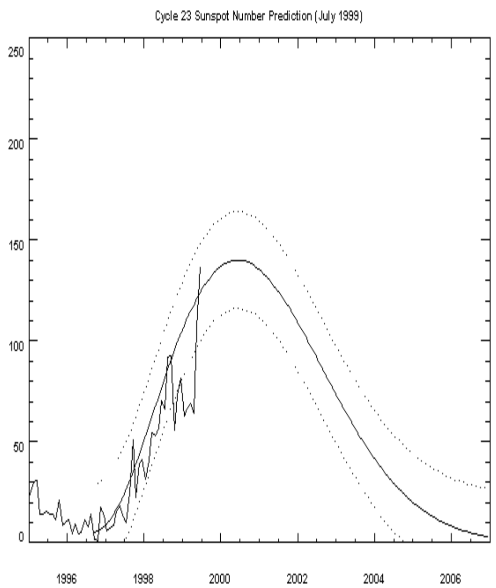

Figure 1. Sunspot number for current solar cycle, predicted and measured (NASA, Marshall SFC, Solar Physics).

One way to resolve the carrier phase ambiguities on the fly (OTF) over distances of hundreds of kilometers between rover and reference stations, is to make double-differenced ionospheric corrections to the rover's data [1]. As shown in this paper, very precise corrections can be made either in real time or in post-processing, using ionospheric information provided by a network of permanent control stations with dual-frequency receivers.

The corrections can be based on different types of ionospheric models [2], [3], [4], [5], [6]. We have used for this paper, in a real-time mode, a two-layer tomographic model. This model has been shown to provide fast and accurate estimates of the ionospheric total electron content (TEC), in particular in regions (close to the geomagnetic equator) or in periods (like the Solar Maximum) with high electron density variability (see assessment in [3]).

2. MAKING THE IONOSPHERIC CORRECTIONS

Figure 2 shows the layout of the steps taken to get the double-differenced ionospheric corrections for the rover GPS receiver.

Three main steps can be emphasized.

(a) The ionospheric model is created using GPS carrier phase data from the reference stations.

As shown in figure 3, the ionosphere is divided into 3-D cells in a Sun fixed reference frame "local time/geodetic latitude" (cell size of 5x2 degrees respectively and height boundaries at 60-740-1420 km). In these cells we assume that the electron density is constant during the filter batch. Let $L1 = \lambda_1 \phi_1$ (where λ1 is the wave length and ϕ_1 the phase corresponding to the L1 frequency), and $L2 = \lambda_2 \phi_2$ (for the L2 frequency).

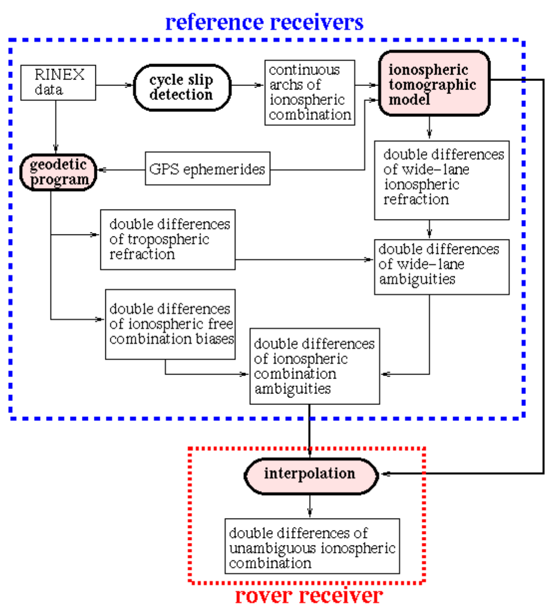

Figure 2. Layout of ionospheric correction procedure.

Then, for each pair satellite-receiver, the frequency dependence of the radio-wave propagation delays in the ionosphere provides the following equations for the phase ionospheric combination *LI=L1-L2* (proportional to the Slant TEC, or STEC) is

$$LI \approx \sum_i \sum_j \sum_k (N_e)_{i,j,k} \Delta s_{i,j,k} + \lambda_1 b_1 - \lambda_2 b_2$$

where *i,j,k* are the indices for each cell corresponding to local time, geodetic latitude and height, $(N_e)_{i,j,k}$ is the corresponding free electron density, and $\Delta s_{i,j,k}$ is the length of the ray path crossing the *i,j,k* cell ($\Delta s_{i,j,k} = 0$ for "dark" cells). Finally, the b_1, b_2 are the ambiguity terms, associated to the wave lengths λ_1 and λ_2, including the instrumental delays.

Similar equations can be written for the ionospheric observable *PI=P2-P1*, based on the pseudo-range P1, P2 measurements. Then the transmitter and receiver code instrumental delays D^T and D_R appear, instead of the ambiguity terms.

Estimating the electron density $(N_e)_{i,j,k}$ from the dual-frequency data measurements is an inverse problem. The parameters of the two-layer model are obtained using a Kalman filter with 10-minute data batch intervals, and assuming the following stochastic behavior for the process noise:

- *Electron density* N_e: random walk with spectral density $dQ/dt = 10^8$ electrons/m^3/hour, in the above mentioned solar-fixed reference frame.
- *Instrumental delays* D^T and D_R: constants
- *Ambiguities* $\lambda_1 b_1 - \lambda_2 b_2$: constant along each arc of continuous data-phase, and white-noise in the cycle-slips.

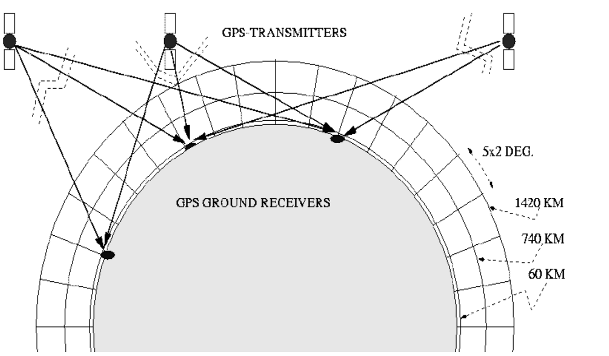

Figure 3. Layout of the two-layer tomographic model adopted to estimate the electron content from reference ground stations

The electron density in the model can be estimated with only carrier-phase data, but the introduction of code data (with an appropriate weight $\sigma_{P1} = 100\sigma_{L1}$) provides more strength in the phase-ambiguities estimation, and allows the estimation of the instrumental delays. After the filter initialization, the solution is mainly driven by the phase data, and hence is practically immune to Anti-Spoofing and code multipath. This is an additional improvement on methods that use pre-aligned phases with the code, or smoothed codes.

(b) Finding the unambiguous ionosphere at fixed sites, with help from a Geodetic program. The coordinates of the permanent control stations are already known at the centimeter-level, in a well-defined reference frame. One can use a geodetic GPS data analysis program (in this case, GIPSY) to estimate, in real time and with centimeter precision, the ionospheric-free combination (Lc) biases and the residual tropospheric refraction left after correcting with a standard atmosphere model. Now one can compute accurate geometric ranges between stations and satellites, corrected for the troposphere and the ionosphere (using the ionospheric model). For distances of a few hundred kilometers, errors in the broadcast ephemeris can be safely ignored.

Since the control stations operate continuously, it is possible to estimate also continuously the ionospheric model, the Lc biases and the tropospheric refraction. When a rover receiver starts to operate, many of these control station quantities should have well-converged sequential estimates ready for immediate use.

The wide-lane ambiguities are found by rounding off the differences between wide lane and refraction-corrected geometric range. It is critical that the combined range error and wide lane noise be less than 43 cm, or half the wide lane's wavelength. One important factor limiting the accuracy of the ionospheric correction is low satellite elevation. Tests we have conducted so far suggest the ionospheric corrections are safe to use down to almost 20 degrees elevation.

From the cm-level estimated biases of the ionospheric-free combinations b_{Lc} and the resolved wide-lane ambiguities N_w (double differenced) it is possible to obtain the *L1, L2* ambiguities N1 and N2. The relevant equations are:

$b_{Lc} = 0.5\ [\lambda_\omega N_w + \lambda_\nu (N1+N2)]$, so
$N1+N2 = \text{Nearest Integer}[(2\ b_{Lc} - \lambda_\omega N_w)/\lambda_\nu]$
$N1 = 0.5\ [N_w+(N1+N2)]$
$N2 = N1-N_w$,
where $\lambda_\omega \approx 86$ cm, $\lambda_\nu \approx 11$ cm, are the wide- and narrow-lanes, respectively.

Finally, one finds the unambiguous double-differenced ionospheric STEC: *LI* = *L1* - *L2* - (λ_1N1- λ_2N2) for the control stations.

(c) Interpolation: In order to find the ionospheric correction for the rover, two schemes have been considered for interpolating to its present location the stations' double-differenced unambiguous *LI*:

- Linear interpolation of the unambiguous ionospheric corrections [1].
- Interpolation driven by the ionospheric model itself: The deviation of the model prediction from the unambiguous ionospheric refraction at all the reference stations is linearly interpolated.

3. RESOLVING AMBIGUITIES ON THE FLY

Two main steps are required in the algorithm used for this study.

(a) Resolving the wide lane. The double-differenced wide lane is first corrected for the ionosphere with the model-interpolated refraction. Then the ionospheric free combination Lc is subtracted. Finally, an estimate of the Lc bias is subtracted as well. This is repeated for every double difference for which there is a reliable ionospheric correction (satellites above 20 degrees in elevation).
The result of this operation is the wide lane ambiguity (in meters), plus carrier phase noise, minus the error in the ionospheric correction and in the estimated Lc bias:

$\lambda_w N_w$+noise(wide lane)-error(ionospheric + Lc bias)
= wide lane - Lc - ion. correction - estimated Lc bias

The wide lane ambiguity, N_w, can be found by rounding off this result to the nearest integer number of wide lane wavelengths. Errors in computed tropospheric refraction, reference station coordinates, and satellite ephemeris, cancel out when subtracting Lc from the wide lane. Assuming the ionospheric correction is sufficiently accurate to resolve the L1 and L2 ambiguities (< 2.7 cm error in *LI*), the main uncertainty is that in the Lc bias. Assuming further that the main sources of uncertainty are normally distributed, then the error in the Lc bias should be less than 1/4 of a wide lane (< 21.5 cm) for the procedure to work well at least 95% of the time.
The Lc bias must be estimated simultaneously with the position of the rover, which, in general, cannot be considered sufficiently well known beforehand.
Other unknowns are errors in tropospheric correction and in broadcast ephemeris. As in real-time, a Kalman filter is used to obtain a joint solution for the present position and all the other unknowns (in post-processing one would go further and use a smoother). The data are dual-frequency carrier phase and, whenever possible, ionosphere-corrected L1 pseudo-ranges
This joint solution has been made using software developed by the first author for precise, long-range kinematic and static positioning [7], [8], [9], modified to apply the ionospheric corrections. This software has been used repeatedly to calculate static and kinematic position with sub-decimeter precision, over distances of more than 1000 km. All the unknowns required for the present work are already part of the observation equations. In particular, the long-range technique involves "floating" the ambiguities, that is to say, using Lc as the main data type, and solving for its biases as "real numbers". The calculation also gives the precision of each estimated Lc bias, to decide when this estimate is precise enough to use safely.

(b) Resolving the L1 and L2 ambiguities. Once the wide lane integer ambiguity N_w is known, one can exploit the following constraint that the corrected, ambiguous ionospheric observable *LI* = *L1* - *L2* must satisfy:

LI- N_1 (λ_1-λ_2)+N_w λ_2 - ionospheric correction

= noise(*LI*)+error(ionospheric correction for *LI*)

From this follows that

N_1 = Nearest Integer[(*LI*- N_w λ_2)/(λ_1-λ_2)],

as long as the sum of noise and ionospheric refraction error in *LI* is less than half $|\lambda_1-\lambda_2|$, or 2.7 cm. Finally:

$N_2 = N_1 - N_w$

Once N_1 and N_2 have been found, the exact Lc bias can be calculated and assimilated in the Kalman filter, as an additional (pseudo-)observation. Since the filter in question is of the usual covariance type, some "noise" uncertainty must be assigned to every observation. In this case, σ = 1 cm has been chosen as a conveniently small value.

The procedure outlined above does not require an integer search as long as the various uncertainties are smaller than the specified bounds. In particular, the effect of data noise has been reduced by using data averages. This has been easy to do, because of a characteristic of the long-range technique, which uses data compression (averaging) to economize time and hard disk scratch space. By trial and error, a data-averaging interval of 2 minutes has been selected. Generally speaking, it takes about three consecutive filter updates with 2-minute carrier phase averages, to make Lc bias estimates exceeding the 20 cm precision needed to resolve the wide lane. (For faster resolution, an integer search could do better, if taking properly into account the various uncertainties.)

No ambiguity is considered resolved unless the absolute value of the corrected, averaged, and presumably unbiased *LI* was less than 1.5 cm. No resolved Lc biases are accepted unless they pass an error null hypothesis test on being assimilated in the Kalman filter. And, while many Lc biases might be satisfactorily resolved every two minutes, only those that are not already known with better than 3 cm-precision are updated with their resolved values.

4. THE BELLKIN TEST OF 23 MARCH 1999

The Institut Cartogràfic de Catalunya (ICC) is deploying CATNET [10], a network of permanent GPS stations covering Catalunya. The core of the network is a subset of three stations located at the vertices of the triangular shape of Catalunya (in the NE corner of Spain, between France and the Mediterranean) and one station located at the center, (figure 4), the sides of the triangle are 200 - 300 km long. All the stations are equipped with high quality GPS receivers and are recording data at 1Hz, although when not needed part of the files are filtered to 15 seconds record interval prior to downloading them to the ICC headquarters.

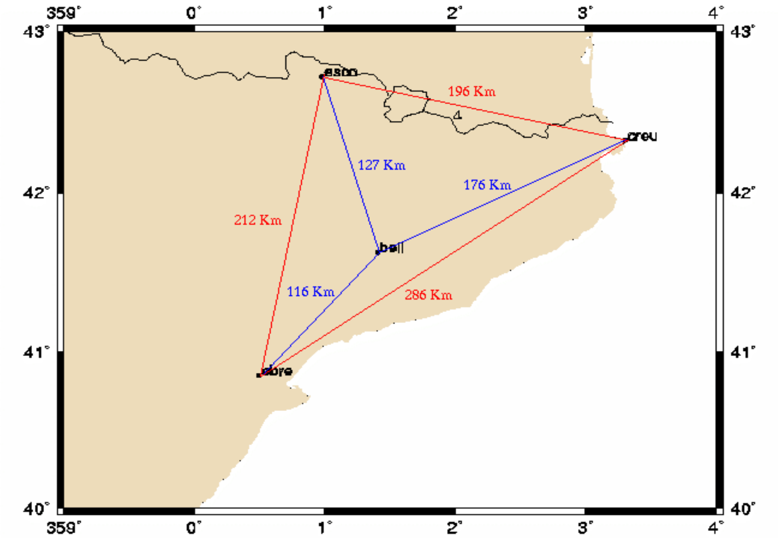

Figure 4. Permanent CATNET stations.

A kinematic test was prepared in order to confirm good preliminary results obtained with static CATNET data taken in 1997. The idea was to build a ionospheric model with the data from the permanent GPS stations located at the vertices of the Catalan triangle (EBRE, CREU, ESCO), and then apply corrections based on this model to the rover receiver. So a place near the central site (BELL) was selected to carry out the test. The test area was located at more than 100 km from the nearest permanent GPS station used to create the ionospheric model, but in the middle of the triangle, making the test well conditioned, as the model was interpolated and not extrapolated. Also, the permanent GPS station BELL was very close (> 3 km) to the test area so it was possible to compute a good reference trajectory using standard OTF kinematic processing techniques. The fact that BELL and the rover receiver were located in the same small area meant that the same ionospheric correction values could be applied to the rover and to the permanent station. So, as an additional check, the data from the static permanent station could be treated as if they were kinematic data, comparing the resulting kinematic positions to the very accurately known position of this site, when the ionospheric model was applied.

On 23 march 1999 the BELLKIN test was carried out using a car with a GPS antenna mounted on the top. The car was driven several times in a loop, and three times the antenna was placed on a tripod just in case a static initialization should be performed. The total length of the test was 2.5 hours, most of them useful except for short periods when obstructions resulted in the rover keeping lock on less than four satellites.

The ionospheric model has been computed with software from gAGE/UPC written by the three middle authors, and using GIPSY for the geodetic calculations needed to resolve the fixed-site ambiguities. The vehicle trajectory has been computed with software developed by the first author, modified to implement the OTF algorithm described in Paragraph 3, and with GeoTeX:TraDer from the ICC.

All crucial calculations were made in "IBM-clone" PC's, running under LINUX.

All double differences used in the calculations were made relative to the EBRE site and (during the actual kinematic test) to satellite PRN 21.

5. RESULTS

5.1 Ionospheric Corrections.

The first step in finding the ionospheric corrections is to determine the wide lane ambiguities for the control sites. Figure 5 shows the percentage of wide lanes resolved successfully in trials conducted every 10 minutes, as a function of the elevation of the lowest satellite. The resolution was deemed correct when the result was the same as the ambiguity calculated from the difference, averaged over several hours, of the phase wide lane and the pseudo-range narrow lane. After processing 6 hours of data from the fixed stations, the determination of the ionospheric TEC and residual tropospheric delay had converged sufficiently. Over the next 6 hours, the ionospheric effect on the wide lane of all new satellites was consistently estimated in 6 minutes or less, with sufficient precision to resolve the wide lane ambiguity in most cases. Given that the station coordinates are known very precisely, the Lc bias can be determined to better than 3 cm in 2 - 4 minutes. So, by the time the wide lane had been resolved, the Lc bias was already known well enough to resolve the L1 and L2 ambiguities.
As shown, the tomographic model allowed many more ambiguities to be resolved at much lower elevations.

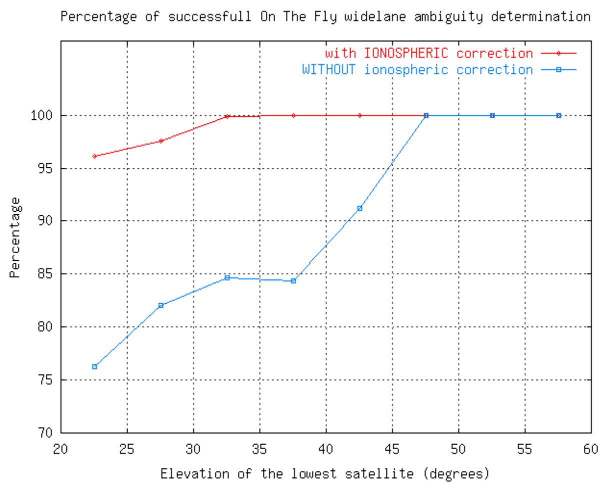

Figure 5. Success in wide lane determination as a function of satellite elevation

The interpolation procedure described in Paragraph 2 was tested comparing the unambiguous ionospheric observable *LI*, double-differenced relative to station EBRE, and satellite PRN 21, and interpolated from the three peripheral control sites to the central site BELL, with the actual observables at BELL (which had had their own ambiguities resolved in static post-processing, for maximum accuracy). Figure 6 shows an example of the percentage of double differences, at ten-minute intervals, for which discrepancy between the interpolated and the actual observable was less than 2.7 cm.
A larger discrepancy is likely to indicate an incorrect resolution of the L1 and L2 ambiguities. By this criterion, with the exception of double-differences involving satellite PRN 31 (which seems to have had unresolved cycle slips), all the *LI* ambiguities should have been obtained correctly nearly 100% of the time using the model-assisted interpolator (after a start-up period of about 30 minutes, in the example shown).
Consequently, this interpolator was used to calculate the corrections for the vehicle, in the kinematic test.

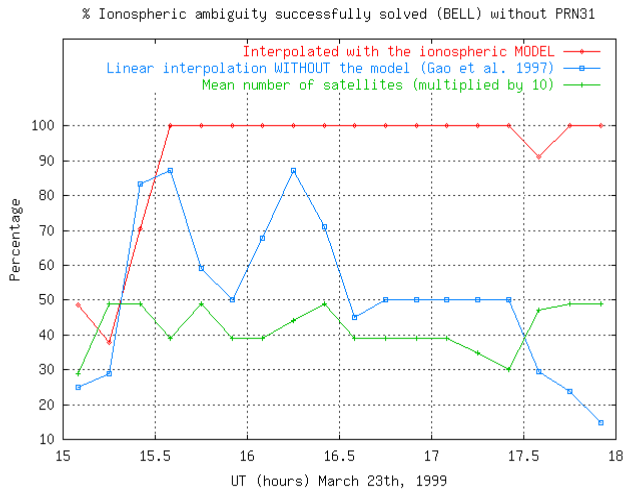

Figure 6. Percentage of ionospheric corrections for BELL with errors < 2.7 cm, for two different interpolators ("Mean Number of Satellites": above 20^o elevation.)

5.2 Kinematic Solutions.

Two OTF kinematic solutions were made, using the ionospheric corrections explained in Paragraph 3: one solution for the car and the other for the fixed site BELL. In each case the actual position was known with centimeter-level precision from independent solutions. The car trajectory had been determined precisely relative to the nearby receiver at BELL, using standard OTF procedures. BELL itself has been very carefully positioned in the IGS frame, as part of the CATNET activity. For a network this size, the relative position of the control sites should be better than 1 cm, so fixing the three outer sites to slightly incorrect values should have virtually no impact on the results. The broadcast ephemeris were used, as in a real-time situation. Present indications are that these ephemeris are precise enough that over 100 - 200 km baselines the effect of their errors

on the computed vehicle position should be a few centimeters, at most, and the effect on individual double differences, less than 2 cm. As in a real-time case, the orbit errors were estimated, along with refraction correction errors, floated Lc biases, and the position of the vehicle.
The a priori uncertainties for the vehicle were σ = 100 m per coordinate. These coordinates were treated as zero-memory, or white noise, error states. So no dynamics were assumed and, with the coordinates almost freely adjusted, this was a true kinematic solution. Calculations were later repeated without solving for the broadcast ephemeris errors, changing the estimated vehicle position by a few cm, as expected.

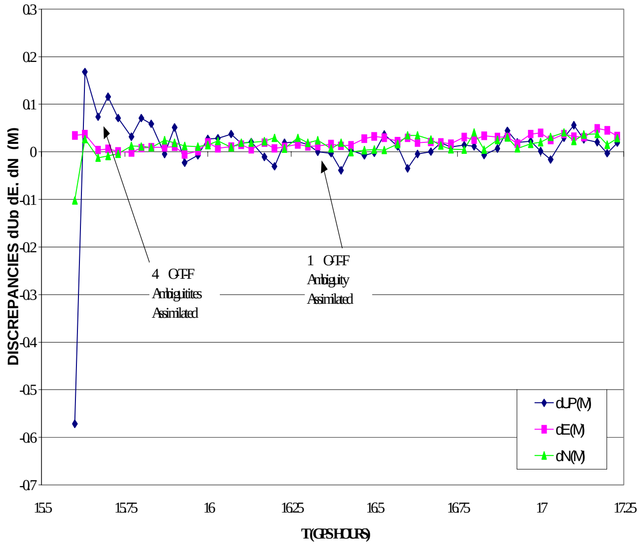

Figure 7. Up, East, and North discrepancies (in meters) between station BELL kinematic OTF position relative to EBRE (116 km away) , and its precise static position. First 4, and then 1 O-T-F ambiguities are assimilated in Kalman filter.

Each OTF solution has been tested in two ways:

(a) Observing the discrepancies between the known position and the kinematic position relative to EBRE, 116 km away. This test only covers attempts to resolve ambiguities when these are actually needed, at the start of new double differences (mostly at the beginning of the trajectory).

The resulting Up, East, and North discrepancies are shown in figures 7 and 8.
Both for the fixed site and the moving car, the discrepancies become less than 10 cm at most 8 minutes after the appearance of new satellites. That is the time it takes to determine the ambiguities.

(b) Comparing the Lc biases calculated using all the resolved N1 and N2 integers (whether assimilated by the filter or not) to their precise post-processed estimates.

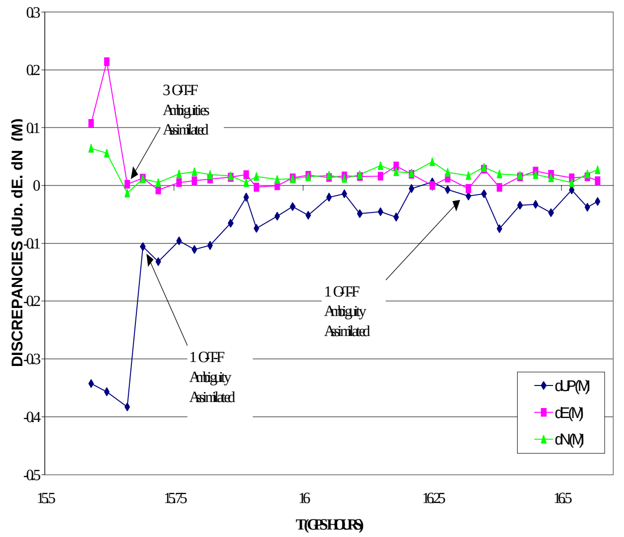

Figure 8. Discrepancies between long-range kinematic OTF position for the car relative to EBRE, and a precise control trajectory (relative to nearby BELL). Filter converges after assimilating first 4 O-T-F ambiguities.

Separate kinematic solutions were made in post-processing, with the known position of the rover as a constraint. (In this particular case, it was enough to constrain the solution for the first two minutes, to get the post-processed position within centimeters of the known position along the whole trajectory.) The precision of the resulting Lc biases is better than 1 cm. Because of the precautions taken, it is very unlikely that the wide lane is resolved with an error of more than one cycle. Then the bias calculated using the erroneous N1 and N2 is unlikely to differ by more than ~5 cm from the true value of the Lc. bias, whatever the errors. The L1, L2 ambiguities have been regarded as successfully resolved if such discrepancy is less than 2 cm.
Table 1 shows the number of trials, or ambiguity resolutions attempted at the end of all the consecutive 2-minute data averaging intervals, both for BELL and for the car. Success in satisfying criteria (a) and (b) above, was 100%

TABLE 1

Kinematic OTF Test Summary

No. Trials	Rover	Success
150	BELL	100%
101	CAR	100%

CONCLUSIONS

There was a combined total of 251 attempts to resolve ambiguities, during an aggregate of more than four hours of kinematic processing. All the attempts seemed successful. However, the level of global geomagnetic activity at the time, with a planetary Kp of 2, was quite moderate, so conditions in the region of the test probably were mild. Additional work with static receivers [11] suggests that effective ionospheric corrections can be calculated under more severe conditions, using the same techniques presented here. To test these ideas further, more experiments with a rover should be conducted in the future.

The ionospheric corrections were calculated in Spain, and the kinematic OTF solutions, in Denmark. The software used for those calculations is an ad-hoc collection of what was readily available. It is quite possible to implement the technique described in this paper in dedicated software for real-world applications, running in an ordinary PC, as all the software used in this research does.

These results show that control networks such as CATNET have the potential to support very precise GPS navigation and surveying work, both in real time and in post-processing, inside areas hundreds of kilometers across.

ACKNOWLEDGEMENTS

This work has been partially supported by the Spanish IGUANA project (CICYT TIC97-0993-C02-01)

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